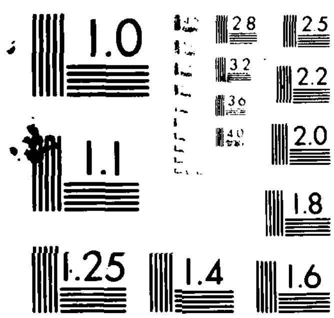
AD-A110 614 NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/G 21/5
EFFECT ON FUEL EFFICIENCY OF PARAMETER VARIATIONS IN THE COST F--ETC(U)
SEP 81 B L DOUGHERTY

UNCLASSIFIED NL

1 of 2



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

LEVEL II

(2)

NAVAL POSTGRADUATE SCHOOL

Monterey, California

AD A110614

DTIC ELECTE FEB 8 1982 H

THESIS

EFFECT ON FUEL EFFICIENCY OF PARAMETER VARIATIONS IN THE COST FUNCTION FOR MULTIVARIABLE CONTROL OF A TURBOFAN ENGINE

by

Barry Lawrence Dougherty

September 1981

Thesis Advisor: D.J. Collins

Approved for public release; distribution unlimited.

82 02 04

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A110 614
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) Effect on Fuel Efficiency of Parameter Variations in the Cost Function for Multivariable Control of a Turbofan Engine	5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; September 1981
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Barry Lawrence Dougherty	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	12. REPORT DATE September 1981
	13. NUMBER OF PAGES 105
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Multivariable Control
F100 Turbofan Engine
Jet Engine Control
Jet Engine Fuel Efficiency

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In modern turbofan engines, variable geometry has been incorporated to improve some off-design performance. Most control designs ignore this variable geometry and use fuel metering as the primary control input.

This thesis investigates the use of variable geometry to control the engine and, thereby, reduce fuel consumption due to transients. Additionally, steady-state trim conditions are altered

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

#20 - ABSTRACT - (CONTINUED)

to reduce the static fuel consumption. The non-linear transient simulation program is used to analyze the steady-state operating condition and develop small perturbation control limitations. Linear models, both large and reduced order, are used in analyzing the effect of controllers on system response. A computer program was generated to reduce a large order linear model to a usable size for control system development.

This analysis shows the reduced-order regime dependent controllers to be viable and to favorably enhance the quest for reducing specific fuel consumption in existing engines.

Approved for public release; distribution unlimited.

Effect on Fuel Efficiency of Parameter Variations in the Cost Function for Multivariable Control of a Turbofan Engine

by

Barry Lawrence Dougherty
Lieutenant, United States Navy
B.S.A.E., United States Naval Academy, 1972

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1981

Author: Barry L. Dougherty

Approved by: Daniel J. Collins
Thesis Advisor

Donald M. Layton
Second Reader

Chairman, Department of Aeronautics

William M. Tolles
Dean of Science and Engineering

ABSTRACT

In modern turbofan engines, variable geometry has been incorporated to improve some off-design performance. Most control designs ignore this variable geometry and use fuel metering as the primary control input.

This thesis investigates the use of variable geometry to control the engine and, thereby, reduce fuel consumption due to transients. Additionally, steady-state trim conditions are altered to reduce the static fuel consumption. The non-linear transient simulation program is used to analyze the steady-state operating condition and develop small perturbation control limitations. Linear models, both large and reduced order, are used in analyzing the effect of controllers on system response. A computer program was generated to reduce a large order linear model to a usable size for control system development.

This analysis shows the reduced-order regime dependent controllers to be viable and to favorably enhance the quest for reducing specific fuel consumption in existing engines.

TABLE OF CONTENTS

I. INTRODUCTION ---------------------------------- 11

II. TURBOFAN ENGINE PERFORMANCE CONSIDERATIONS ------ 14

A. FLIGHT REGIME FACTORS ---------------------- 14

B. TURBOFAN ENGINE PERFORMANCE MEASUREMENTS ---- 17

C. ENGINE CONTROL DEVICES --------------------- 19

D. ENGINE MODELING ---------------------------- 21

E. CONTROL DEVICE EFFECTIVENESS --------------- 25

III. CONTROL DEVELOPMENT ---------------------------- 34

A. LINEAR OPTIMAL CONTROL THEORY -------------- 34

B. CONTROL SYSTEM MODELS ---------------------- 36

C. REDUCTION OF LINEAR MODELS ----------------- 41

D. REDUCED ORDER MODELS ----------------------- 45

E. VERIFICATION OF REDUCED ORDER MODELS -------- 52

IV. ANALYSIS OF CONTROL SYSTEMS -------------------- 55

A. COST FUNCTION WEIGHING FACTORS ------------- 55

B. REDUCED ORDER SIMULATION ANALYSIS ---------- 66

C. FULL-ORDER SIMULATION ANALYSIS ------------- 68

V. CONCLUSIONS AND RECOMMENDATIONS ---------------- 73

APPENDIX A -- 75

APPENDIX B -- 95

LIST OF REFERENCES --------------------------------- 103

INITIAL DISTRIBUTION LIST -------------------------- 105

LIST OF TABLES

II-1 Engine States ---------------------------------- 24

II-2 Engine Inputs and Outputs -------------------- 26

II-3 Input Perturbations -------------------------- 31

II-4 Summary of Derivatives ----------------------- 32

III-1 Operating Point Eigenvalues ----------------- 46

III-2 Reduced Model Comparisons ------------------- 47

III-3 Reduced Model Eigenvalues ------------------- 49

III-4 Fifth Order Model ---------------------------- 50

III-5 Third Order Model ---------------------------- 51

IV-1 Case Definition ------------------------------ 58

IV-2 Actuator Limitations ------------------------- 60

LIST OF FIGURES

II-1	Flight Envelope Limitations --------------------	15
II-2	Flight Regimes ----------------------------------	16
II-3	Engine Cutaway ----------------------------------	18
II-4	Compressor Stage Flow --------------------------	22
II-5	Change in SFC vs Control Inputs ----------------	28
II-6	Change in Thrust vs Control Inputs -------------	29
II-7	Change in Turbine Inlet Temperature vs Control Inputs --------------------------------	30
III-1	Model Comparison in Frequency Domain -----------	53
IV-1	SFC vs Control Inputs --------------------------	62
IV-2	Thrust vs Control Inputs -----------------------	63
IV-3	Fan Speed vs Control Inputs --------------------	64
IV-4	Compressor Speed vs Control Inputs -------------	65
IV-5	Third Order vs Fifth Order Control Response ----	67
IV-6	Fifth Order vs Full Order Control Response -----	69
IV-7	Third Order vs Full Order Control Response -----	70
IV-8	Full Order Implementation ----------------------	71

TABLE OF SYMBOLS

SFC	Thrust specific fuel consumption
M_a	Total engine airflow
M_f	Main burner fuel flow
U_e'	Hot stream velocity
U_e''	Cold (fan) stream velocity
P_{0x}	Total pressure at station x
T_{0x}	Total temperature at station x
R	Universal gas constant
Fn	Net engine thrust
A_j	Nozzle jet area
CIVV	Inlet variable guide vanes
RCVV	High compressor variable vanes
X	The state variable vector
Y	The output vector
U	The input vector
x$	The $ element of the X vector
y$	The $ element of the Y vector
u$	The $ element of the U vector
F	The plant matrix
Fr	The reduced-order plant matrix
G	The input gain matrix
Gr	The reduced-order input gain matrix
H	The output matrix
Hr	The reduced-order output matrix

D	The feed-forward matrix
Dr	The reduced-order feed-forward matrix
K	The feedback gain matrix
R_1	The state weighting matrix
R_2	The input weighting matrix
P	The Riccati solution vector
T	The transpose matrix
Z	The redefined state vector
A	The output control matrix
A_y	The output weighting matrix
J	The cost function
β	The fan bypass ratio
η	The efficiency factor
γ	The ratio of specific heats

ACKNOWLEDGMENTS

I would like to thank all of the people, without the help of whom, this thesis could not have been completed.

I would especially like to thank Professor D. J. Collins, my thesis advisor, for his help and guidance; Mr. Neil Harvey of the W. R. Church Computer Center; and, most especially, my wife and daughter for their support and patience during my tenure at the Naval Postgraduate School.

I. INTRODUCTION

High performance, military aircraft are operationally tasked to conduct missions throughout their designed performance envelopes. To accomplish these missions, the aircraft must operate efficiently within its flight regime. The typical jet engine is designed for operation at a single specified altitude and Mach number combination, e.g., 30,000 feet at M = 0.9. This design point will give inherently good crûise performance and is particularly well-suited to commercial transport operations. This engine design method forces the user into accepting off-design performance in typical military applications. Engine performance needs to be optimized to efficiently conduct all assigned missions, e.g., air combat maneuvering, ground attack, supersonic interception and carrier operations. Adaptive digital control methods can be incorporated within the engine to accomplish this mission dependent optimization.

Modern control methods allow the designer to develop a control system to regulate every parameter of the parent system. He has been aided by computer simulations of nonlinear systems and linear approximations of those systems. The desired control system can be developed parallel to or ahead of the controlled system.

The state-of-the-art turbofan engine utilizes variable geometry, in such forms as variable incidence inlet guide

vanes, adjustable exhaust nozzle area and airflow bleed-off. Hydromechanical controls have been used in the past to improve some elements of engine performance through positioning these variable engine components. Electronic controls, such as were introduced in the Pratt & Whitney F100-PW-100 turbofan engine, allow improved scheduling of the variable devices, again improving the engine performance. Studies have been done on the F100 engine to optimize its performance at a point other than its design point [1].

Originally, the onboard microprocessing capability of an aircraft was intended for weapon system improvement and integration. Further computing equipment was added as part of the engine control to supervise the hydromechanical control system and detect faults. The F100 engine introduced an electronic control which allowed use of digital control techniques incorporating the hydromechanical devices as implementors rather than controllers. Digital device improvements continue to expand the available computing power available, making flight control and engine control integration a distinct possibility. As more software space becomes available to the engine controller, regime adaptive control will be the accepted standard in military applications.

This thesis looks at using modern, multivariate control logic in designing flight-regime dependent control for a typical military turbofan engine, the Pratt & Whitney F100. Specifically, the attempt is made to improve thrust specific fuel consumption, i.e., fuel economy, in the high altitude

cruise environment. Such improvement in fuel economy is especially important in the conservation-conscious decades ahead.

II. TURBOFAN ENGINE PERFORMANCE CONSIDERATIONS

A. FLIGHT REGIME FACTORS

The ability of a military aircraft to perform an assigned mission is determined by its flight regime. Figure II-1 is a typical flight regime for a modern, high-performance military aircraft.

The confines of the flight regime are dictated by numerous aerodynamic and propulsive factors. A well designed aircraft will incorporate an engine having operating boundaries in excess of those of the airframe. This design technique allows for a factor of safety in actual operation as well as compensate for installation and equipment aging losses.

At the low speed, low altitude region of the envelope, engine performance is judged in the ability to produce sufficient thrust for take-off and landing and to provide rapid engine acceleration in the missed approach or balked landing. The extreme upper region of the flight envelope requires the engine to produce maximum thrust while not exceeding turbine temperature limitations.

Figure II-2 is a mission delineated flight regime for the supersonic tactical fighter aircraft. The desired mission performance from this chart must be interrelated with the aerodynamic and propulsion limitations of Figure II-1. An adaptive control system can be designed to schedule optimum performance of the airframe and engine throughout the flight

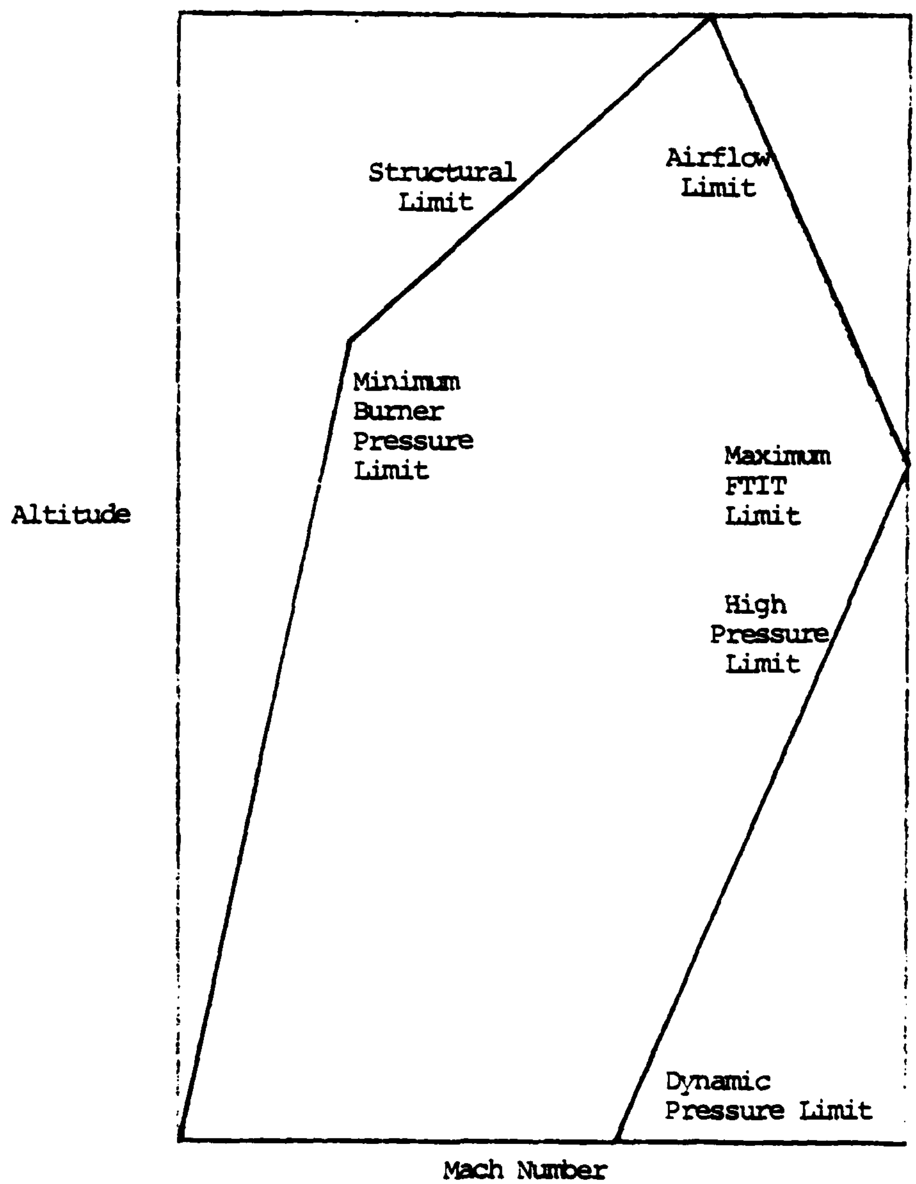


Figure II-1. Flight Envelope Limitations

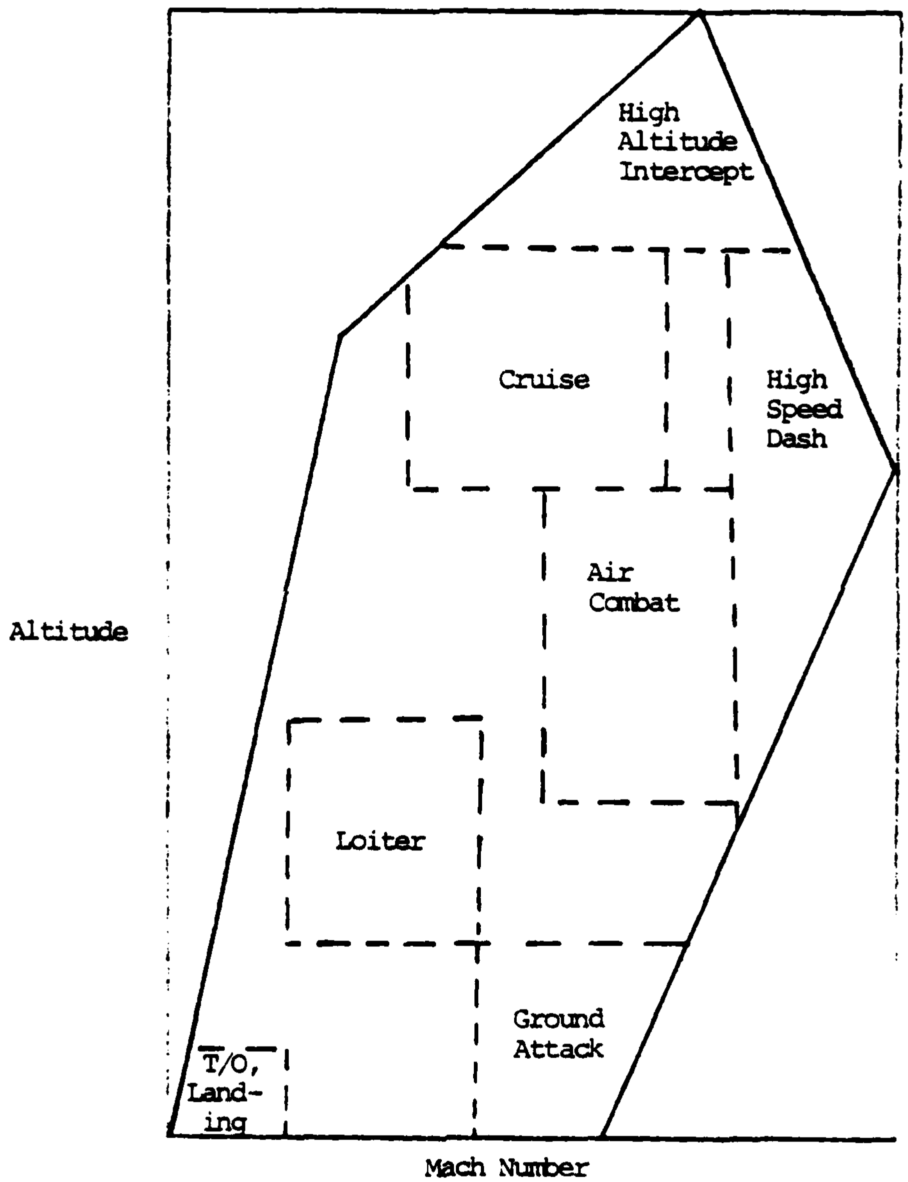


Figure II-2. Flight Regimes

regime. Northrup Corporation is involved with a U.S. Air Force project to implement an adaptive flight control system in an aircraft such as the F-16. The same type of approach can be applied to engine control and integrated into an aircraft control system to optimize the total package.

B. TURBOFAN ENGINE PERFORMANCE MEASUREMENTS

The primary factor with which to judge engine performance is the engine's ability to convert fuel to thrust efficiently. This measurement, thrust specific fuel consumption, SFC, is a function of the airflow through the engine, M_a, the bypass ratio, b, and the fan and core exhaust velocities, U_e'' and U_e', also termed cold and hot stream velocities,

$$SFC = \frac{M_a}{M_f}[(1+f)U_e' + \beta U_e'' - (1+\beta)V] \qquad \text{(II-1)}$$

where M_f is the fuel flow, f is the fuel-to-air ratio and V is the flight velocity. Figure II-3 shows the F100-PW-100 engine in cross section with reference labels.

The engine designer fixes the airflow and bypass ratio of the engine. The exhaust velocities are functions of the exhaust temperatures and pressures, which are controllable through fuel metering and variable geometry.

$$U_e' = \left\{\frac{2\gamma_1}{\gamma_1-1} R\, T_{06}\, \eta\left[1 - \left(\frac{p_a}{p_{06}}\right)^{\gamma_1 - 1/\gamma_1}\right]\right\}^{1/2} \qquad \text{(II-2)}$$

$$U_e'' = \left\{\frac{2\gamma}{\gamma-1} R\, T_{08}\, \eta\left[1 - \left(\frac{p_a}{p_{08}}\right)^{\gamma - 1/\gamma}\right]\right\}^{1/2} \qquad \text{(II-3)}$$

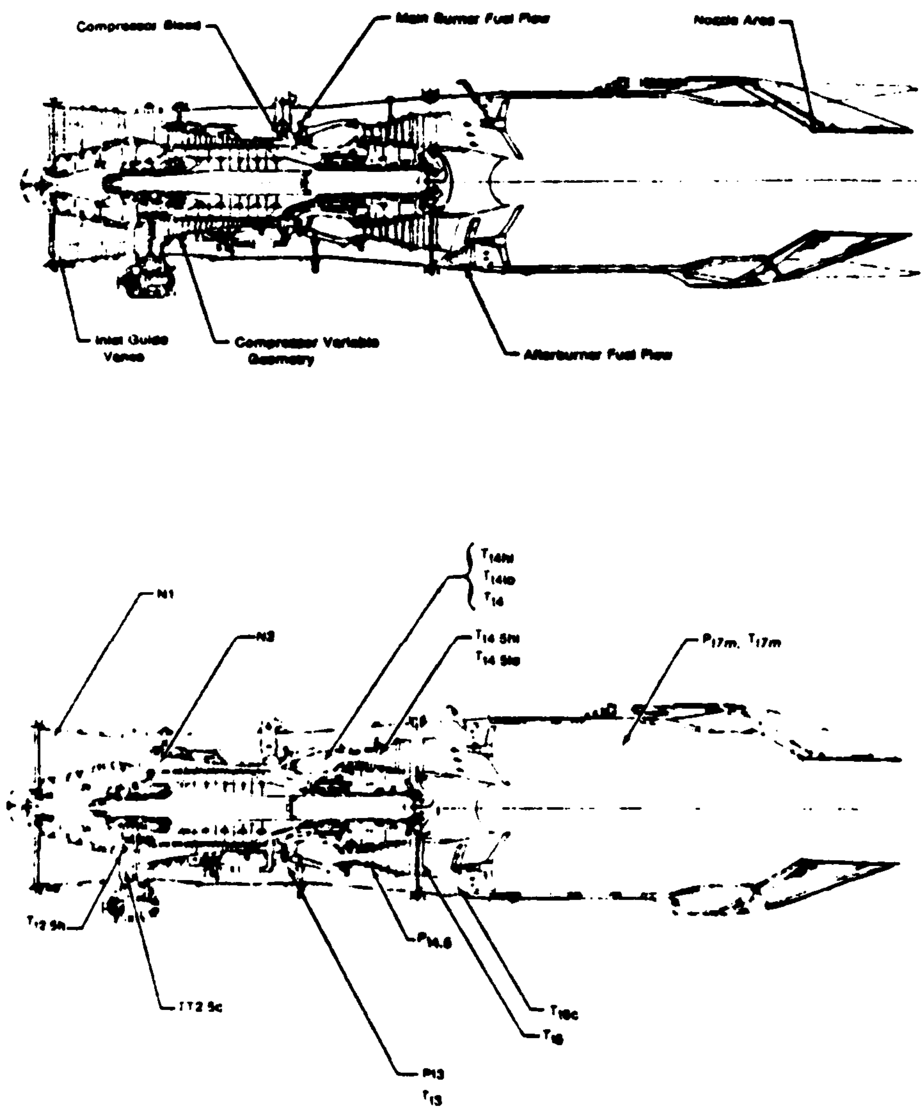


Figure II-3. Engine Cutaway

where γ and γ_1 are the respective ratios of specific heats, T_{06} and T_{08} the respective exhaust total temperatures, η is the exhaust nozzle efficiency, and p_a/p_{06} and p_a/p_{08} the respective ratios of free stream ambient pressure to discharge total pressure.

The standard bypass fan will produce a fixed pressure ratio and exhaust temperature, producing an exhaust velocity in the cold stream in excess of the flight velocity. This increases the engine's thrust without altering the hot stream through the engine core (ignoring energy lost in driving the fan). The advanced, low-bypass fans that reintroduce the cold flow into the augmentor, as in the F100, increase the augmentor efficiency and during non-augmented operation, allow for increasing U_e'' through the use of the variable exhaust nozzle.

Studies show that Thrust Specific Fuel Consumption, SFC, can be decreased through increased turbine inlet temperature and compressor pressure ratio. Both of these variables are designed into the engine at the engine design point. Engine control hardware can be designed to maintain the turbine inlet temperature, FTIT, and pressure ratio, R_c, at the optimum or best achievable level during off-design operations.

C. ENGINE CONTROL DEVICES

The mechanical devices available for engine control are the hydromechanical fuel control, the variable exhaust nozzle and variable incidence inlet guide vanes or stators. The

first device has the effect of increasing the fuel-to-air ratio and temperatures in the turbine and exhaust sections of the engine. The combined effect of increasing f and U_e' is increased thrust, but does not necessarily improve SFC due to the corresponding increase in the fuel flow. The variable area exhaust nozzle has been used for the last two decades in most high performance military aircraft. This device increases the hot section exhaust velocity, U_e', thus increasing thrust and maintaining near ideal pressure in the exhaust section of the engine.

The latest mechanical devices are the variable incidence blading concepts. The first use of these devices was to achieve near design mass flow rate through the compressor during the engine start cycle. These variable stators would be rotated to a starting configuration until the engine reached a given rotational speed and then be driven to a fixed normal operating position.

The full advantage of variable geometry blading comes in the optimizing of off-design performance. The single stage pressure ratio is found to be

$$\frac{P_{03}}{P_{01}} = 1 + \eta \frac{U_r^2}{RT_{01}}[1 - \frac{C_z}{U_r}(\tan b_2 + \tan a_1)] \qquad \text{(II-4)}$$

where η is the stage efficiency, U_r is the rotational velocity of the blading, R is the Universal Gas Constant and T_{01} the total temperature at the front face of the stage. At the

design condition, the factor (tan b_2 + tan a_1) is minimized, thus providing the best pressure ratio. The angle b_2 is the relative flow direction upon leaving the rotor portion of the machine; a_1 is the angle at which the flow impinges the rotor and is a function of the stator angle (see Fig. II-4). Controlling the stator angle increases the stage pressure ratio in the early stages and increases the compressor (or fan) performance over a wide range of operations.

In the F100-PW-100 engine, four mechanical devices are used, fuel control, variable exhaust nozzle, inlet guide vanes with moveable trailing edge and moveable stators in the first three compressor stages. The exhaust nozzle maintains nozzle area, expansion ratio and boattail drag, simultaneously, near optimum. The inlet guide vanes are used ahead of the fan to improve inlet distortion tolerance, improve fan efficiency and enhance engine acceleration performance. The variable stators improve starting and high Mach number characteristics.

D. ENGINE MODELING

It is not economically practical to build an engine and then conduct experimentation to determine its performance. Modern technology, in both digital and hybrid computers, allows the design engineer to build a simulation model of the engine and predict the system response well in advance of hardware manufacture. Several generic computer simulations are in use, these allow non-specific engine performance analysis. Additionally, a computer model is generated for each new development engine by the engine manufacturer.

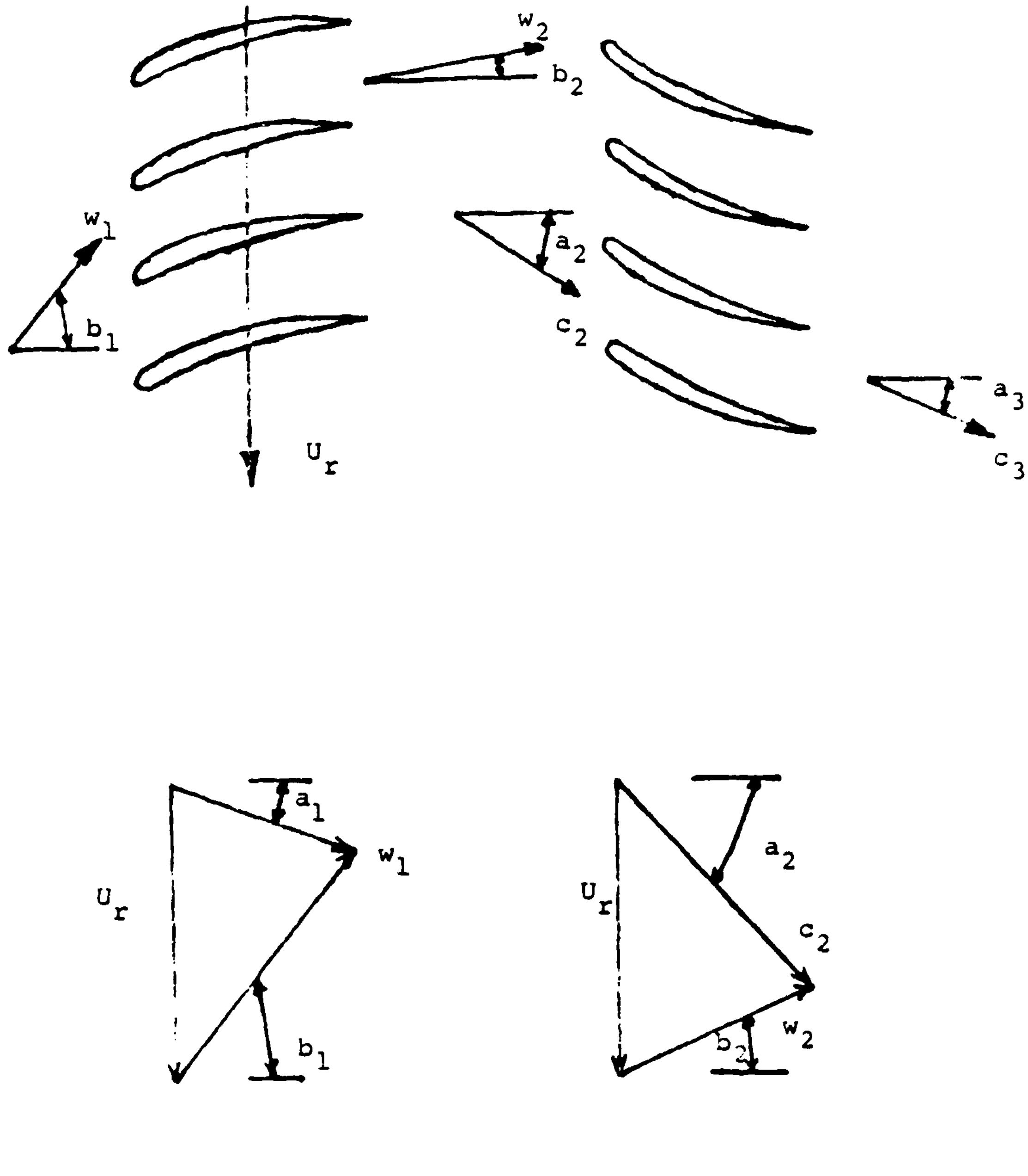


Figure II-4. Compressor Stage Flow

Szuch, [2], provides a detailed description of current engine modeling techniques. The Pratt & Whitney F100-PW-100 engine has been modeled with both a real-time hybrid computer [3], and a digital transient simulation [4]. Both programs have distinct advantages and applications. Although the digital program is not real-time, it provides a large volume of data and is easily accessed and maintained. A copy of the F100-PW-100 Transient Engine Simulation Deck (CCD 1103.3.0) was obtained from the YF100 Special Project Office of the Aeronautical Systems Division of the U.S. Air Force at Wright-Patterson Air Force Base, Ohio. This program was modified to be compatible with the Naval Postgraduate School IBM 370/3033 computer system.

Designing a controller based on the non-linear digital simulation alone is extremely difficult, if not impossible. Miller and Hackney [5], developed high order linear approximations, at steady state operating points, from the non-linear simulation program. The linear system was modeled in the classical state variable form

$$\dot{X} = FX + GU \qquad \text{(II-5)}$$

$$Y = HX + DU \qquad \text{(II-6)}$$

The 16 system states, listed in Table II-1, were chosen to coincide with the states measured in the non-linear simulation. These states include 2 rotational speeds, 3 internal total

TABLE II-1

Engine States

x1	--	Fan speed, (N1), in RPM
x2	--	Compressor speed, (N2), in RPM
x3	--	Compressor discharge pressure, (Pt3), in psia
x4	--	Interturbine volume pressure, (Pt4.5), in psia
x5	--	Augmentor pressure, (Pt7m), in psia
x6	--	Fan inside diameter discharge temperature, (Tt2.5h), in Rankine
x7	--	Duct temperature, (Tt2.5c), in R
x8	-	Compressor discharge temperature, (Tt3), in R
x9	--	Burner exit fast response temperature, (Tt4hi), in R
x10	--	Burner exit slow response temperature, (Tt4lo), in R
x11	--	Burnerexit total temperature, (Tt4), in R
x12	--	Fan turbine inlet fast response temperature, (Tt4.5hi), in R
x13	--	Fan turbine inlet slow response temperature, (Tt4.5lo), in R
x14	--	Fan turbine exit temperature, (Tt5), in R
x15	--	Duct exit temperature, (Tt6c), in R
x16	--	Duct exit temperature, (Tt7m), in R

pressures and 11 total temperatures. The inputs to the system are the fuel flow, the 3 variable geometry elements previously discussed, and the customer bleed air percentage. The outputs include the net thrust, the engine total airflow, turbine inlet temperature and the stall margins. Additionally, two fan exit pressure ratios are determined for some of the operating points. The inputs and outputs are listed in Table II-2.

The technique used by Miller and Hackney perturbed each state, xi, slightly while holding the other states and inputs constant. This allows calculation of the deviations caused by that state in the other states and outputs. These deviations then form the F and H matrices of equations (II-5) and (II-6). Perturbing each input, ui, in the steady state configuration determine the G and D matrices. The sampling time was set at 7 milliseconds.

The linear models obtained are very good approximations of the non-linear system but are often too complex and/or do not contain the most convenient parameterization to be used in a practical design. A reduction of model order and, perhaps, augmentation of the linear model is generally required prior to using the model for control design.

E. CONTROL DEVICE EFFECTIVENESS

Before designing an engine control, one must determine the effect each input has on the system and the responsiveness of each input. The five control inputs listed in Table

TABLE II-2

Engine Inputs

u1 -- Main burner fuel flow, lb/hr

u2 -- Nozzle jet area, ft**2

u3 -- Inlet guide vane position, deg

u4 -- High compressor variable vane position, deg

u5 -- Customer bleed flow, %

Engine Outputs

y1 -- Net thrust, lb

y2 -- Total engine airflow, lb/sec

y3 -- Turbine inlet temperature, R

y4 -- Fan stall margin

y5 -- Compressor stall margin

y6 -- Fan exit delta p ratio (test data)

y7 -- Fan exit delta p ratio (theory)

II-2 are used in controlling the F100 engine. The customer bleed air percentage, u5, is essentially constant in steady state operations and can therefore be ignored in the control development. A study could be made to determine the merit in extracting additional bleed air from the engine as a primary control, and dumping that portion not required by the customer systems.

The intent of this control development is to minimize the specific fuel consumption of the engine. At the same time, only small variations in turbine inlet temperature, y3, and total engine airflow, y2, are desirable and no change in thrust is allowable. One method might employ using exhaust nozzle area, u2, and fan inlet guide vanes, u3, inputs to minimize SFC at the trim condition and then compensate for thrust loss by using fuel to balance the equation.

Figure II-5 plots the change in specific fuel consumption due to incremental changes shown in Table II-3 in the inputs u1, u2, and u3; input u4, the compressor variable guide vanes, exhibit no influence on SFC. Proportional derivatives, i.e., slopes were determined by making a linear approximation to the curves. It is clear in the figure that nozzle area and inlet guide vane inputs (u2 and u3) have the most substantial effect on SFC.

Figure II-6 plots the net engine thrust, y1, versus the control input changes. Again u2 has the greatest effect and u1 and u3 contribute about equally to thrust variations. It is also noted that a linear combination of u1 and u3 can be

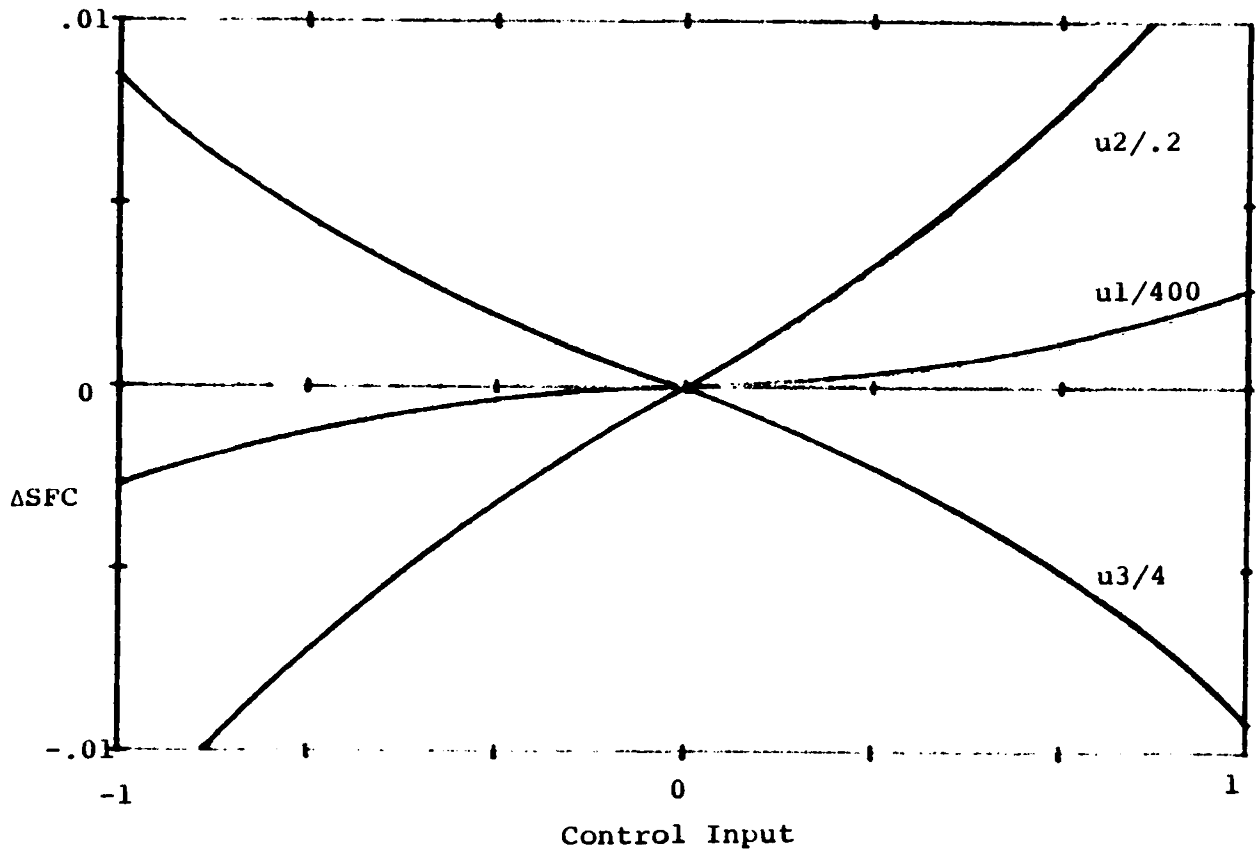


Figure II-5. Change in SFC vs Control Inputs

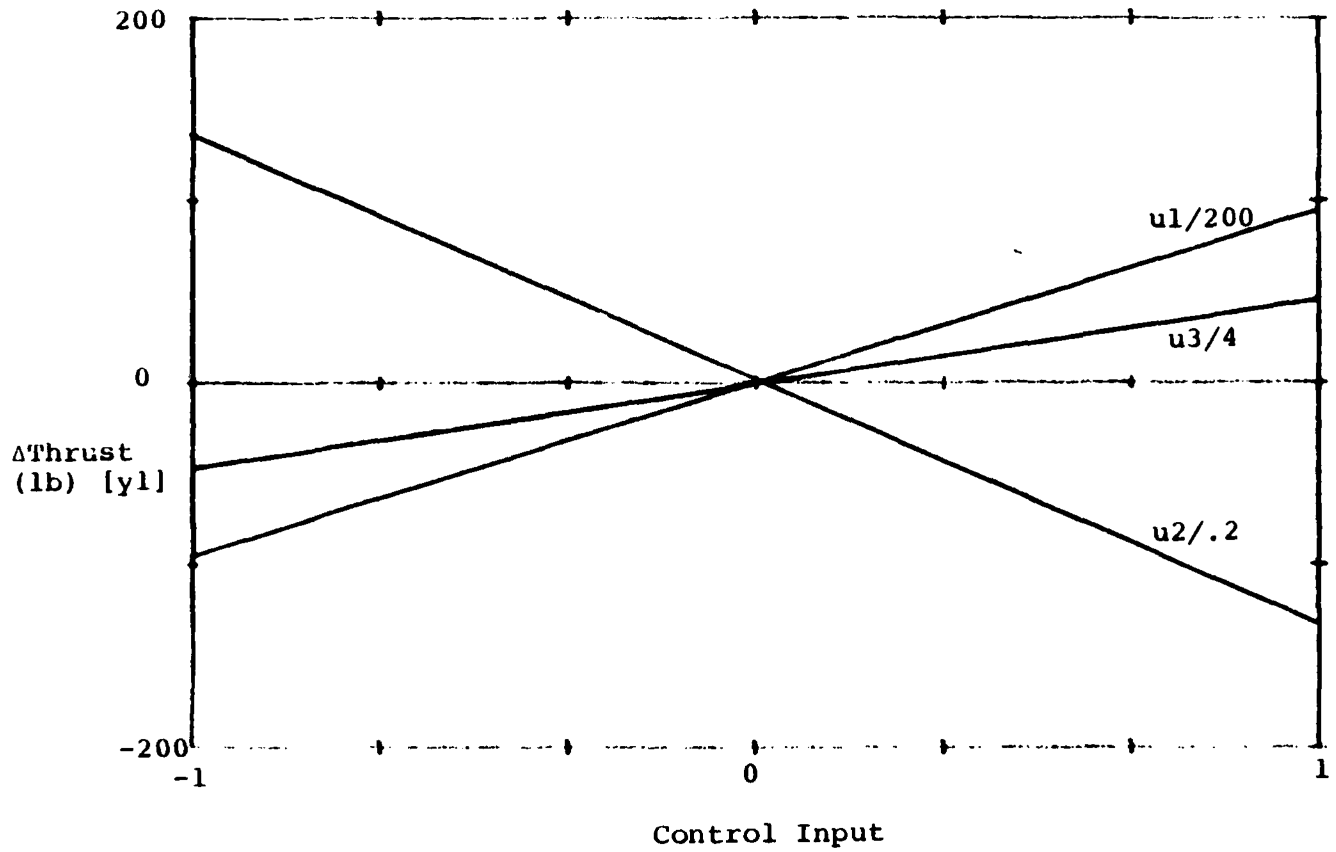


Figure II-6. Change in Thrust vs Control Inputs

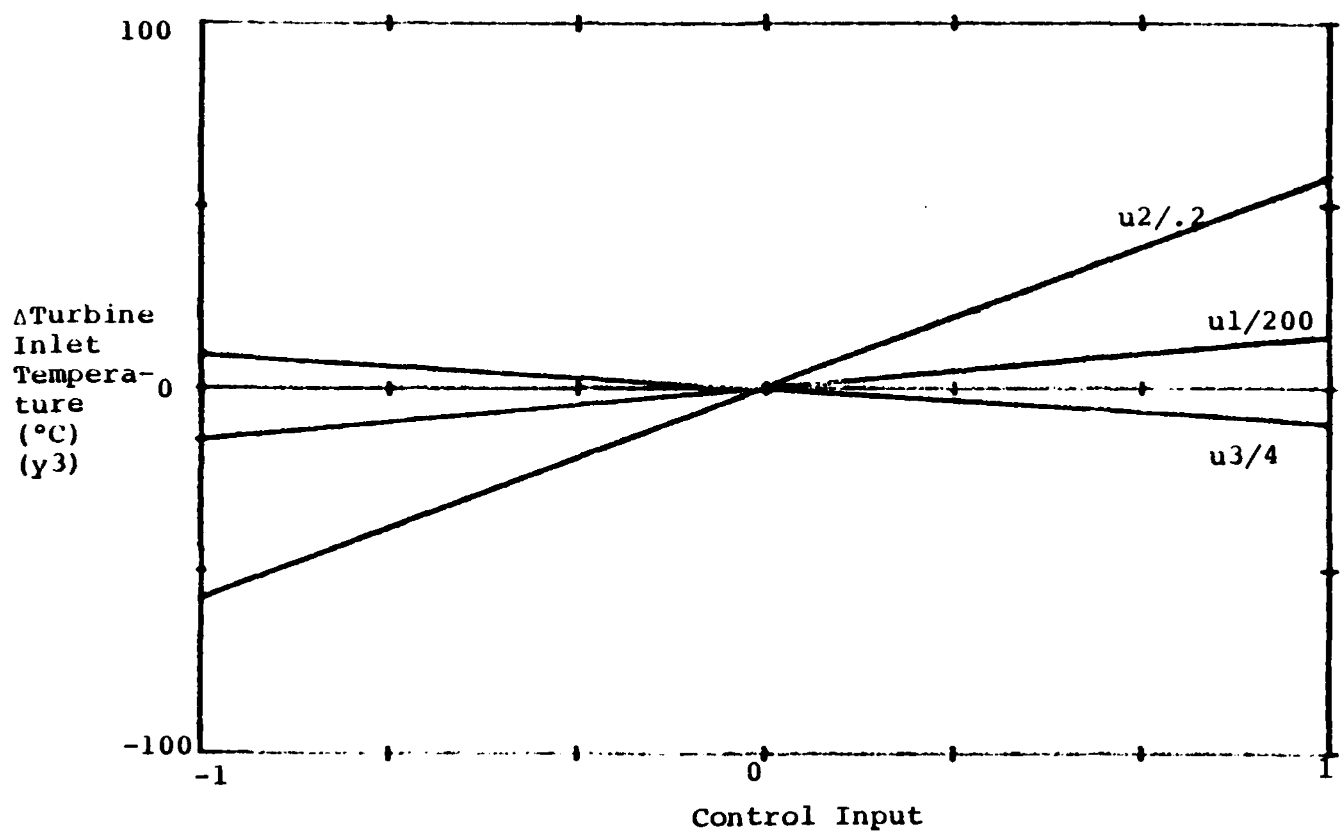


Figure II-7. Change in Turbine Inlet Temperature vs Control Inputs

TABLE II-3

Input Perturbations

The following perturbations are used throughout this investigation. They represent the largest expected value, or physical limitation placed on each control input.

Fuel Flow, u1	± 200 lb/hr
Nozzle Area, u2	± 0.2 sq. ft.
Inlet Guide Vanes, u3	± 4 degrees
Compressor Vanes, u4	± 4 degrees
Bleed Air, u5	0.0 percent

used to offset the thrust change caused by a change in u2. This is important in meeting the zero net thrust change design requirement. Figure II-7 shows the effect the inputs have on turbine inlet temperature, y3. Again, u2 is the dominant control input. These figures clearly demonstrate the need for minimizing the exhaust pressure mismatch with ambient conditions.

Table II-4 is a summary of the proportional derivatives extracted from the preceding figures. Using these derivatives the following relationships are found.

$$d(SFC) = 2.27E{-}05\, d(u1) + .0655 d(u2) - .00092 d(u3) \qquad (II\text{-}7)$$

$$d(y1) = .9325\, d(u1) - 1351.7 d(u2) + 18.6275 d(u3) \qquad (II\text{-}8)$$

TABLE II-4

Summary of Derivatives

Input	SFC	Thrust (y1)	FTIT (y3)
Fuel Flow, u1	2.27E-5	.9325	.12126
Nozzle Area, u2	.0655	-1351.7	283.085
Inlet Guide Vanes, u3	-.00092	18.6275	-2.1641

U4 was found to have no effect on the parameters of interest.

$$d(y3) = .12126d(u1) + 283.085d(u2) - 2.1641d(u3) \quad (II-9)$$

If a decrease in u2 of -.2 square feet and an increase of +4 degrees in u3 are applied, a change of -.017 occurs in SFC. The fuel flow, u1, is used to balance the thrust equation. To accomplish this the fuel is reduced by 370 pounds per hour, further decreasing specific fuel consumption. The total effect of these three inputs at the trim condition is a 3.5% reduction in SFC, no thrust change and a 110 degree decrease in the turbine inlet temperature.

By requiring that thrust level not change, the SFC performance is compared under identical operating conditions. Each input can be studied separately to determine its effect on the outputs and on SFC. To remain in the small perturbation regime inputs should not be varied more than 10% of their steady-state values. Combinations of inputs are then chosen to effect the most desirable improvement in SFC and minimize the effect on engine performance.

This technique has shown improvement in the specific fuel consumption at the operating point chosen in the studies of Reference 1. The use of the existing variable geometry to improve the fuel efficiency at the high altitude cruise operating point chosen for this investigation will be discussed in a later section.

III. CONTROL DEVELOPMENT

A. LINEAR OPTIMAL CONTROL THEORY

Given the linear system,

$$\dot{X} = FX + GU \qquad \text{(III-1)}$$

consider a linear control law defined by

$$U = -KX + U_0 \qquad \text{(III-2)}$$

The original system can be modified to

$$\dot{X} = (F - GK)X + GU_0 \qquad \text{(III-3)}$$

The response of the modified system, as well as its eigenvalues, is determined by the effect of the gain matrix, K. For a low order system, the control gains could be found by trial-and-error methods and applied to the system to determine if the response was acceptable. This method is impractical for large order systems.

Optimal control is achieved by minimizing a cost function, J, which is defined as

$$J = \frac{1}{2} \int [(X^T R_1 X) + (U^T R_2 U)]dt \qquad \text{(III-4)}$$

where R_1 and R_2 are symmetric weighing matrices; R_1 is non-negative definite and R_2 is positive definite. A third portion can be added to (III-4) to provide terminal state weighing,

$$X^T(t_1)P_1X(t_1) \tag{III-5}$$

where P_1 is a non-negative definite symmetric matrix.

It is shown by Kwakernaak and Sivan [8], that the optimal input, $U_0(t)$, based on the cost function is given by

$$U_0(t) = (R_2)^{-1}G^TP(t) \tag{III-6}$$

thus the knowledge of p(t) solves the regulator problem. A 2n linear system is formed with the optimal system behavior, $x_0(t)$, and the adjoint variable, p(t).

$$\begin{bmatrix} \dot{X}_0(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} F & -GR_2^{-1}G^T \\ -R_1 & -F^T \end{bmatrix} \begin{bmatrix} X_0(t) \\ p(t) \end{bmatrix} \tag{III-7}$$

where

$$p(t) = P(t)X_0(t) \tag{III-8}$$

with

$$K(t) = R_2^{-1}(t)G^T(t)P(t) \tag{III-9}$$

P(t) is a symmetric non-negative definite matrix that satisfies the Riccati equation

$$-\dot{P}(t) = R_1(t) - P(t)G(t)R_2^{-1}(t)G^T(t)P(t) + P(t)F(t) + F^T(t)P(t) \qquad \text{(III-10)}$$

with the terminal condition of

$$P(t_1) = P_1 \qquad \text{(III-11)}$$

Under these conditions, as the terminal time, t1, approaches infinity, the control law asymptotically approaches a unique, stable steady state condition. Thus, the matrix Riccati equation, (III-10), can be rewritten

$$0 = R_1 - PGR_2^{-1}G^TP + PF + F^TP \qquad \text{(III-12)}$$

and

$$K = R_2^{-1}G^TP \qquad \text{(III-13)}$$

Many numerical methods are available to solve the matrix Riccati equation for the steady-state control gains. Some of these are covered by Kwakernaak and Sivan [6]. This thesis employed a computer solution, OPTSYS4, developed by Hall and Bryson [7], at Stanford University. A representative output is included in Appendix B.

B. CONTROL SYSTEM MODELS

At this point, an analysis of control models is made to establish the form of the controller to be applied. A

comparison of small and large perturbation controls must be made. Take the linear system

$$\dot{X} = FX + GU \qquad \text{(III-14)}$$

$$Y = HX + DU \qquad \text{(III-15)}$$

This system can be made closed-loop by incorporation of a control law

$$U = -KX \qquad \text{(III-16)}$$

K can be a time-varying matrix or constant as required by the system. A more general control law would be

$$U = U_m + C_x(X - X_m) + \int C_y A(Y - Y_0)dt \qquad \text{(III-17)}$$

where A is the diagonal matrix whose diagonal terms are 1 or 0 determining which outputs are used in formulating the input. C_x and C_y are again matrices associated with the states and the outputs, respectively. Additionally define

$$U_m = U_0 + R_u t \qquad \text{(III-18)}$$

$$X_m = X_0 + R_x t \qquad \text{(III-19)}$$

the 0 subscript indicates trim position/value; R_u and R_x are functions of u and x associated with time. For small perturbations the time varying terms, R_u and R_x, can be ignored and

equation (III-17) can be rewritten

$$\delta U = C_x \delta X + \int C_y A \delta Y \, dt \qquad \text{(III-20)}$$

Dropping the delta terminology and taking the derivative of this and using equation (III-16), yields

$$- K\dot{X} = C_x \dot{X} + C_y AY \qquad \text{(III-21)}$$

combining terms

$$(-K-C_x)\dot{X} = C_y AY \qquad \text{(III-22)}$$

Substituting, using the system description of equations (III-14) and (III-15) one obtains

$$(-K-C_x)(FX+GU) = C_y A(HX+DU) \qquad \text{(III-23)}$$

and solving for C_x,

$$C_x = -K-C_y A(H-DK)(F-GK)^{-1} \qquad \text{(III-24)}$$

and using all outputs ($A = I$) and equally weighing them ($Cy = I$) one has

$$C_x = -K-(H-DK)(F-GK)^{-1} \qquad \text{(III-25)}$$

However, if the outputs are ignored (A = 0), i.e., considering the case where small changes are made in the output vector, then

$$C_x = -K \tag{III-26}$$

as in the simplified control law of equation (III-16).

This approach does not apply to the large perturbation case. In large perturbation analysis, the time varying terms cannot be ignored. The mean values of u and x are as in equations (III-18) and (III-19). R_u and R_x are the time functions that describe the path that the mean values follow over the period of the large perturbation. The control law becomes

$$\delta U - R_u t = C_x(\delta X - R_x t) + \int C_y A \delta Y \, dt \tag{III-27}$$

Again, if all outputs are used and equally weighed, then taking the derivative

$$-R_u - \dot{R}_u t = C_x \dot{X} - C_x R_x - C_x \dot{R}_x t + C_y AY \tag{III-28}$$

and combining terms

$$-C_x(F-GK)X + R_x C_x + \dot{R}_x C_x t = K(F-GK)X + R_u + \dot{R}_u t + (H-DK)X \tag{III-29}$$

If we set

$$R_x C_x = R_u \qquad \text{(III-30)}$$

and

$$\dot{R}_x C_x = \dot{R}_u \qquad \text{(III-31)}$$

in effect, assuming a linear relationship between the states and inputs. Then

$$-C_x = -K - (H-DK)(F-GK)^{-1} \qquad \text{(III-32)}$$

as in equation (III-25) for the small perturbation case.

An adaptive control system removes the need to model the large perturbation response. Linear models can be developed, as described in Section II.E, to define the large perturbation case as a series of linear relations. The onboard digital computer then schedules the control gains based on the linear models as the flight conditions pass through the model transition points. The rapidity of response inherent to the digital computer removes the requirement for modeling the total dynamic system, rather, only the dominant response model for the selected operating point. The reduced-order linear model is far easier to handle in formulating the control system and is derived to match the dominant system characteristics.

C. REDUCTION OF LINEAR MODELS

DeHoff and Hall [8], present a method of reducing the order of the linear system based on the dominance of the states and the crosscoupling of the states as observed through eigenvector analysis. The required order of reduction is determined by the number of dominant states appearing in the desired control bandwidth. For the F100 study, DeHoff and Hall chose a control bandwidth of 0 to 10 rad/sec, corresponding to the primary control device (fuel flow actuator). Their operating point (static, sea-level, intermediate power) dictated that the control be designed to modulate thrust, providing maximum thrust for take-off performance. At another operating point a different parameter may dictate the control design and, thus, the primary control actuator and control bandwidth.

The method of decomposition involves finding the eigenvector transform matrix, T, such that

$$XT = T\Lambda \tag{III-33}$$

where Λ is the diagonal eigenvalue matrix. Then, by defining an alternate state vector, Z, where

$$X = TZ \tag{III-34}$$

the original system may be written in modal coordinates as

$$\dot{Z} = \Lambda Z + T^{-1}BU \tag{III-35}$$

If a control bandwidth is determined, the eigenvalues falling in that range are the only ones that need to be included in the initial reduced order model. The matrix may be reordered by constructing a matrix Q, which has a 1 in the position corresponding to the eigenvalue to be included (column) and the reordered position (row) of that eigenvalue. Thus, if eigenvalue 5 of the original Λ matrix was to be reordered to the first position, a 1 would be placed in the (1,5) position of the Q matrix. Now, the reordered matrix

$$\Lambda' = Q\Lambda \tag{III-36}$$

is incorporated in the linear model

$$Q\dot{Z} = \Lambda' Z + QT^{-1}BU \tag{III-37}$$

letting

$$Z' = QZ \tag{III-38}$$

$$\dot{Z}' = \Lambda' Q^{-1} Z' + QT^{-1}BU \tag{III-39}$$

and

$$X = TQ^{-1}Z' \tag{III-40}$$

A matrix R can be found such that

$$X' = RX = TZ' = \begin{bmatrix} X_1 \\ --- \\ X_2 \end{bmatrix} \tag{III-41}$$

This X' vector is the reordered state vector, where the elements contained in the subvector X_1 are those states associated with the eigenvalues in the control bandwidth and X_2 contains all other states.

The linear system may be rewritten

$$\dot{X}' = RFR^{-1}X' + RGU \qquad \text{(III-42)}$$

$$Y = HR^{-1}X' + DU \qquad \text{(III-43)}$$

Now define

$$F' = RFR^{-1} \qquad \text{(III-44)}$$

$$G' = RG \qquad \text{(III-45)}$$

$$H' = HR^{-1} \qquad \text{(III-46)}$$

and

$$F' = \left[\begin{array}{c|c} F_{11} & F_{12} \\ \hline F_{21} & F_{22} \end{array}\right] \qquad \text{(III-47)}$$

$$G' = \left[\begin{array}{c} G_1 \\ \hline G_2 \end{array}\right] \qquad \text{(III-48)}$$

$$H' = \left[\begin{array}{c|c} H_1 & H_2 \end{array}\right] \qquad \text{(III-49)}$$

If the states in X_1 truly model the full order system, then $\dot{X}_2$ will be essentially zero and

$$\dot{X}_1 = F_{11}X_1 + F_{12}X_2 + G_1U \qquad \text{(III-50)}$$

$$0 = F_{21}X_1 + F_{22}X_2 + G_2U \qquad \text{(III-51)}$$

Solving for X_2 in equation III-51, one has

$$X_2 = - F_{22}^{-1}F_{21}X_1 + F_{22}^{-1}G_2U \qquad \text{(III-52)}$$

This is incorporated into the first equation

$$\dot{X}_1 = (F_{11}-F_{12}F_{22}^{-1}F_{21})X_1 + (G_1-F_{12}F_{22}^{-1}G_2)U \qquad \text{(III-53)}$$

and the output equation becomes

$$Y = (H_1-H_2F_{22}^{-1}F_{21})X_1 + (D-H_2F_{22}^{-1}G_2)U \qquad \text{(III-54)}$$

The reduced order model can be written as

$$\dot{X}_1 = FrX_1 + GrU \qquad \text{(III-55)}$$

$$Y = HrX_1 + DrU \qquad \text{(III-56)}$$

where

$$Fr = F_{11} - F_{12}F_{22}^{-1}F_{21} \qquad \text{(III-57)}$$

$$Gr = G_1 - F_{12}F_{22}^{-1}G_2 \tag{III-58}$$

$$Hr = H_1 - H_2F_{22}^{-1}F_{21} \tag{III-59}$$

$$Dr = D - H_2F_{22}^{-1}G_2 \tag{III-60}$$

This technique was used to develop the program REDUCR2, documented in Appendix A. REDUCR2 was used to develop the reduced order models used in the control development portion of this thesis. Sample input and output of REDUCR2 are listed in Appendix A.

D. REDUCED ORDER MODELS

Three reduced order models were generated with the REDUCR2 computer program (Appendix A). The criteria for selecting the desired states was that used by DeHoff and Hall, a control bandwidth of 0-10 Hertz. The eigenvalues of the full 16th order system were found using the OPTSYS4 computer program and then associated to the system states using modal analysis. Table III-1 gives the results of this modal analysis. The five states associated with the control bandwidth are fan speed (x1), compressor speed (x2), augmentor pressure (x5), burner exit temperature (x10), and fan turbine inlet temperature (x13).

A fifth order model was generated using these states. It was noted that only three of these states (the two rotational speeds and the pressure) were dynamically significant, i.e.,

TABLE III-1

Operating Point Eigenvalues

State	Physical Property	Eigenvalue	
X4	P04.5	-540.23	
X3	P03	-156.432	
X11	T04	-49.69 ± j13.99	
X16	T07		
X9	T04Hi	- 54.12	
X12	T04.5Hi	- 42.61	
X7	T02.5C	- 22.59	
X14	T05	-21.58 ± j2.51	
X15	T06C		
X6	T02.5H	- 18.62	
X8	T03	- 14.49	
X5	P07	- 7.35	
X1	N1	- 3.60	control band- width
X13	T04.5LO	- 1.98	
X2	N2	- 1.31	
X10	N04LO	- .647	

TABLE III-2

Reduced Model Comparisons

	Model #1	Model #3	Model #2	Full Order
X1	173.51	175.89	175.89	173.41
X2	125.10	126.86	126.86	125.02
X5	.326	.325	.325	.326
X10	4.85			4.86
X11		53.51		53.48
X13	3.98			3.98
X16		23.04		23.03
Y1	94.48	174.8	97.70	156.99
Y3	11.84	53.51	9.3	53.48

represent energy storage in the form of torque and pressure. A third order model was generated using these three states and a comparison made. In this initial analysis these two models produced virtually identical response with respect to the key outputs thrust (y1) and turbine inlet temperature (y3), see Table III-2. The OPTSYS4 program was used to determine the transfer functions associated with these two outputs for each model. It was found that the fifth order model contained zeros near the poles at -1.960 and -.651, the eigenvalues associated with the two temperatures in the fifth order model. This caused these two poles to have no effect on the response of the model. Thus the third order system sufficiently models the control bandwidth.

In both models, the turbine inlet temperature (y3) response did not closely model that of the full order system. Therefore, a third model was formed using the three states of the third order model plus the conjugate pair of poles representing the turbine inlet temperature (x11) and exit temperature (x16). This model gave excellent thrust and temperature response (y1 and y3, Table III-2) due mainly to the inclusion of the turbine inlet temperature as a system state. Table III-3 lists the three models, their states and associated eigenvalues.

Although the conjugate pair of eigenvalues lies outside the control bandwidth, their inclusion markedly improves the output correspondence with respect to the full order system, see Table III-2. Further analysis will use the third order

TABLE III-3

Reduced Model Eigenvalues

	Model #1	Model #3	Model #2
States	X1	X1	X1
	X2	X2	X2
	X5	X5	X5
	X10	X11	
	X13	X16	
Eigen-	-7.175	-51.65 ±	-6.947
Values	-3.392	j7.14	-3.282
	-1.418	- 1.388	-1.389
	- .651	- 6.909	
	-1.960	- 3.265	

TABLE III-4

Fifth Order Model

THE REDUCED PLANT MATRIX IS

-3.050920D+03	2.472570+00	-3.589750+02	8.091320D+00	9.183240-02
1.224290-01	-1.627110+00	4.105960+01	4.289780D+00	3.022930-01
2.728570-03	-2.333140-03	-7.427640+00	-1.169500-02	-9.462420-03
4.131430+00	-5.385390+00	-1.625740+03	-5.385260+01	-2.791070+00
-1.076980+00	2.231960+00	1.146700+03	1.908720+01	-4.889670+01

THE REDUCED CONTROL DISTRIBUTION MATRIX IS

-3.544490-02	-1.408040+02	-9.330150+01	2.35968D+01	-1.83786D+04
-4.409450-01	-2.72213D+01	7.830630+00	-1.095970+01	-9.971110+03
1.752640-02	-3.764650+01	1.211340-01	-7.846500-02	-1.193150+02
1.980160+01	3.163530+02	4.938510+01	-7.681320+01	4.833000+04
-1.890950+00	7.270000+01	-3.043570+01	2.886740+01	7.003520+03

THE REDUCED OUTPUT DISTRIBUTION MATRIX IS

1.928140-01	1.074980-01	1.875340+02	1.208830+00	7.079620-02
6.460660-03	-9.944220-07	-9.154280-03	-5.672210-05	-1.931620-05
0.0	0.0	0.0	1.000000+00	0.0
1.253510-05	1.351230-05	-3.126620-02	-1.390050-04	-4.847580-05
-4.887040-05	1.479580-04	1.016000-03	-5.168060-05	1.449070-05
1.639760-05	-5.035470-05	-3.756300-02	-1.730800-04	-5.956100-05
7.241500-06	-8.125470-07	-1.049280-02	-4.944220-05	-1.668660-05

THE REDUCED FEEDFORWARD MATRIX IS

-1.037660-01	-3.140490+01	-4.020290+03	-3.991000+00	-9.300000+01
-1.092890-04	-2.277060-01	3.466250-01	4.912000-03	-1.314000+01
0.0	0.0	0.0	0.0	0.0
8.620910-05	-7.016040-03	1.831660-02	5.683000-07	-4.671000-01
-1.507840-05	5.898030-03	6.995830-01	-2.745000-03	2.704000-01
1.079410-04	6.266830-03	1.410850-01	-6.403000-04	-4.560000-01
2.925660-05	1.147810-03	4.634340-02	-3.642000-05	-1.298000-01

TABLE III-5

Third Order Model

```
THE REDUCED PLANT MATRIX IS
 -2.338693D+00  1.601100D+00 -6.254560D+02
  4.521490D-01 -2.055580D+00 -8.752910D+01
  1.762600D-03 -1.721000D-03 -7.175860D+00

THE REDUCED CONTROL DISTRIBUTION MATRIX IS
  3.123490D+00 -9.107590D+01 -8.523820D+01  1.118800D+01 -1.075200D+04
  1.144630D+00 -1.725080D+00  1.174380D+01 -1.707580D+01 -6.082560D+03
  1.231590D-02 -3.774800D+01  1.127050D-01 -6.207530D-02 -1.340950D+02

THE REDUCED OUTPUT DISTRIBUTION MATRIX IS
  2.856160D-01 -1.333460D-02  1.511350D+02
  6.456180D-03  4.571820D-06 -7.629180D-03
  7.631480D-02 -1.003380D-01 -3.078140D+01
  1.550630D-06  2.714530D-05 -2.754180D-02
 -5.270190D-05  1.532370D-04  2.772520D-03
  2.926630D-06 -3.337410D-05 -3.291650D-02
  3.338770D-06  4.040280D-06 -9.161720D-03

THE REDUCED FEEDFORWARD MATRIX IS
  3.415580D-01 -3.029840D+01 -2.931490D+03 -3.991000D+00 -9.300000D+01
 -1.318290D-04 -2.277530D-01  2.877980D-01  4.912000D-03 -1.314000D+01
  3.623730D-01  9.304760D-01  8.723760D+02  0.0           0.0
  3.085500D-05 -7.132820D-03 -1.263990D-01  5.683000D-07 -4.671000D-01
 -3.231660D-05  5.846190D-03  6.615080D-01 -2.745000D-03  2.704000D-01
  3.999940D-05  6.121220D-03 -3.871970D-02 -6.403000D-04 -4.560000D-01
  9.625010D-06  1.106130D-03 -4.861250D-03 -3.642000D-05 -1.298000D-01
```

model (model #2) and the fifth order augmented model (model #3). These models are listed in Tables III-4 and III-5.

E. VERIFICATION OF REDUCED ORDER MODELS

The reduced order model must be sufficiently representative of the original system for the designed control to provide the desired full system response. Weinberg and Adams [9], used the F100 non-linear simulation to develop a linear 17th order system at the static, sea-level point at partial power settings. Their reductions to 5th and 3rd order and similar reduction to 5th order by DeHoff [10], from Miller and Hackney's 16th order model show excellent correlation to the non-linear simulation.

The operating point chosen for consideration in this thesis is 30,000 feet, Mach = 0.9 and power level angle of 67 degrees. This corresponds to a near optimum cruise point and is a condition frequently encountered in routine operations. Section III.D lists the states and associated eigenvalues at this operating point and discussed several reduced order models. The dominant eigenvalues were considered to be those which affected the models fast response and had dynamic significance to the engine's physical characteristics. Again, this implies a control bandwidth of 0-10 rad/sec, however, other models, using eigenvalues outside the control bandwidth, were considered in determining the best reduced model for deriving the control laws.

One method to analyze model correspondence is to use the Bode plot across the control bandwidth. Figure III-1 is a

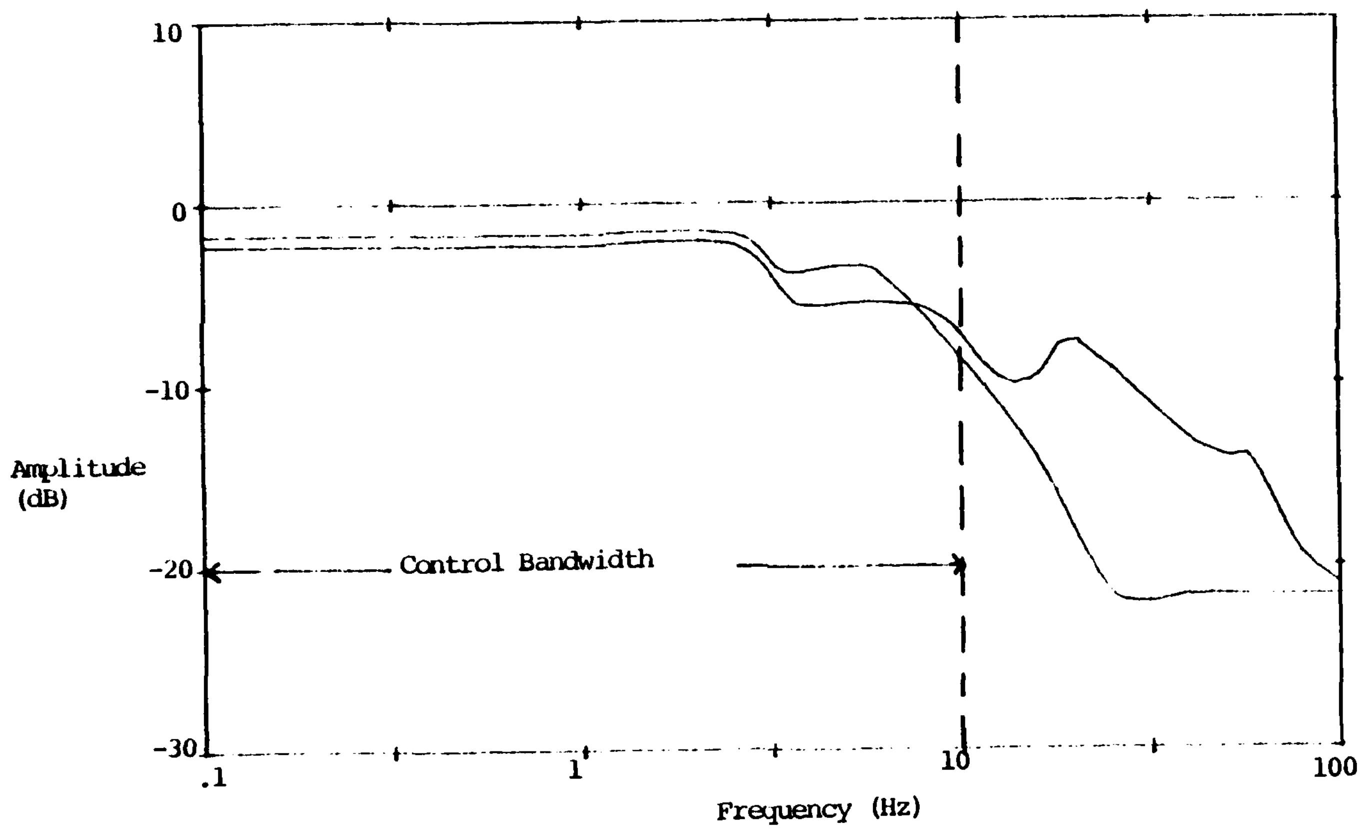


Figure III-1. Model Comparison in Frequency Domain

Bode comparison of the DeHoff reduced model and the Miller and Hackney 16th order model at static, sea-level, intermediate power for the 0 to 10 rad/sec control bandwidth. It is clear that through the 0-10 Hertz bandwidth the two models show very close correspondence; this similarity breaks down in the higher frequency region, but the response time of the eigenvalues outside the control bandwidth is considered to be too short to affect transient behavior. The phase response, not shown, follows the same trends.

IV. ANALYSIS OF CONTROL SYSTEMS

A. COST FUNCTION WEIGHING FACTORS

The control law for the engine regulating system,

$$U = U_m + C_x(X-X_m) + \int C_y A(Y-Y_0)\, dt \qquad \text{(IV-1)}$$

can be reduced as shown in Section III.B to

$$U = U_m - KX \qquad \text{(IV-2)}$$

when considering small perturbations and ignoring the outputs from the system. In most circumstances, the U_m, the trim condition, would be considered as zero for steady state analysis. The linear models developed by Miller and Hackney [5], set all inputs to zero at the steady-state operating point for which the model is defined.

The K matrix is the solution to the optimum regulator problem discussed in Section III.A. The OPTSYS4 computer solution used in this thesis uses the cost function

$$J = \frac{1}{2} \int [Y^T A_y Y + U^T R_2 U] dt \qquad \text{(IV-3)}$$

where A_y is the weighting matrix for the output vector and replaces the need for the R_1 matrix in equation (III-7).

Kirk [11], discusses the effect that weighting matrices have on system performance. In general, the larger the

magnitude of the matrix element the faster its associated parameter (system state, input or output) stabilizes. An additional factor in determining the magnitude of the diagonal matrix elements is the magnitude of their associated parameters. Given the cost function

$$J = \frac{1}{2} \int X^T R_1 X \, dt \qquad \text{(IV-4)}$$

where

$$R_1 = \begin{bmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{bmatrix} \; ; \quad X = \begin{bmatrix} x1 \\ x2 \end{bmatrix} \qquad \text{(IV-5)}$$

The cost function then becomes

$$J = \frac{1}{2} \quad [(\frac{x1}{a})^2 + (\frac{x2}{b})^2] \, dt \qquad \text{(IV-6)}$$

The desire is to minimize the cost function, J, with respect to time. Thus, taking the time derivative and setting it equal to zero

$$(\frac{x1}{a})^2 + (\frac{x2}{b})^2 = 0 \qquad \text{(IV-7)}$$

This is the solution which produces a minimum cost. The values of a and b are the largest expected values of x1 and

x2, respectively, and represent an equal effect on the cost function by each state. Should either a or b be decreased, the optimal control developed would have to increase the control input to the other state to minimize the cost function. A nominal fluctuation limit of ±5% will be considered as a small perturbation in the initial control analysis.

Designing a control to minimize the thrust specific fuel consumption (SFC) of the engine requires limiting the thrust and the fuel flow fluctuations. The weighting matrix elements corresponding to these two parameters are, therefore, to be increased with respect to the other parameters. Looking at only the small perturbation response, it is noted that very little change in the total engine airflow (y3) is expected and, as before, there is no change in customer bleed air requirements (u5). These two parameters can be neglected, as their response characteristics have no effect on the desired SFC response. To demonstrate the effect of the different element magnitudes on the control gain matrix, the OPTSYS4 program was run for the 3 cases delineated in Table IV-1. Case I was developed to allow each output and input, with the exception of the two pressure ratios (y6 and y7) and the bleed air percentage (u5), to effect equal contribution to the cost function. The diagonal elements of both A_y and R_2 normalize the squared value to the order of one. Case II uses all of the same elements, except the thrust (y1) and the fuel flow (u1) are weighted more heavily, to the magnitude of ten. This

TABLE IV-1

Case Definition

A_y and R_2 are diagonal matrices, only the significant elements are listed.

Case I

A_y = (.0001 1 .0001 100 100 1e-8 1e-8)

R_2 = (.0001 100 .01 .01 1e-8)

Case II

A_y = (.01 1 .0001 100 100 1e-8 1e-8)

R_2 = (.01 100 .01 .01 1e-8)

Case III

A_y = (1 1 .0001 100 100 1e-8 1e-8)

R_2 = (1 100 .01 .01 1e-8)

heavier weighting should reduce the fuel input to the system and moderate the thrust fluctuations as the control returns the system to the steady state condition. Case III increases the weighting of y1 and u1 to achieve a normalized magnitude of 100. This extremely heavy weighting should eliminate fuel from the control and reduce thrust variations to absolute minimum. The whole intention of the controller design is to damp out state perturbations without using fuel or altering the thrust level, i.e., making no change in specific fuel consumption to effect control.

These three cases were implemented on the two reduced order models and simulations run for a perturbation of -50 RPM in fan speed (x1). The fifth order model returned the fan speed to 95% of steady state in .5 seconds using Case I, but increased fuel flow by 10 pounds per hour initially and required an increase in fuel consumption of .2% over the half second period. For Case II, the fifth order model returned to 95% of the steady state fan speed in approximately .3 seconds, while using almost 80% less fuel than the Case I controller. This 80% reduction in fuel consumption indicates that the increased weighting of Case II has accomplished exactly what was desired. Case III was run for the fifth order model and, although virtually no fuel was used in the control, the system experienced oscillations in fan speed of ±8 RPM through the 3 second simulation run. Additionally, the Case III controller caused rapid changes of ±4 degrees in the compressor guide vanes (u4), the regulatory limit imposed for the simulation, and also large fluctuations in the inlet guide vanes (u3). One would achieve results of similar magnitude for each of the cases assuming a +1% perturbation of all states in the reduced-order model. Case I is too fuel dependent and causes large changes in u3, Case III also requires large changes in U3 that could cause a slew rate request larger than that available from the control actuator.

In modeling the control, the actuator and physical limitations of each control must be considered and appropriate

limitations placed on the control law to avoid overshoots. Table IV-2 gives the actuator limits and maximum rates applicable to the F100 engine [4]. A stepping limit of .01 seconds is assumed in determining the rate limits and fuel flow fluctuations are limited to ±10% of the steady-state value. No compensation is allowed for hysteresis or non-linear dynamics of the actuators; these are considered minimal in the small perturbation case, but would need to be incorporated for large perturbation control modeling.

TABLE IV-2

Actuator Limitations

Input	Maximum	Minimum	Rate Limit
Fuel flow	16300 lb/hr	450 lb/hr	15800 lb/hr/sec
Nozzle area	6.4 sq. ft.	2.8 sq. ft.	3.6 sq.ft./sec
Inlet guide vanes	0 deg.	-40 deg.	48 deg/sec
Compressor vanes	4 deg.	-40 deg.	40 deg/sec

It is evident that the Case III weighting is not acceptable because of the oscillations and the rapid, large amplitude fluctuations in the guide vane positions (u3 and u4). Case I is also inappropriate for the specific fuel consumption minimizing control since it uses fuel as the primary control input. For these reasons, Case II is chosen as the weighting for all control modeling to be implemented in later sections.

Once the integral portion of equation (IV-1) and the feedback gains are determined, the constant portion, U_m, must be

analyzed. In most situations this term would be neglected. The steady state analysis is now performed to determine if the specific fuel consumption can be reduced by making small changes to the trim position of the variable geometry. An analysis of control changes versus SFC, as was done at the static sea-level condition in Section II.D, must again be conducted. A minimum SFC obtainable relates to certain input values in U_m which is incorporated into the control law, improving the steady-state performance and controlling the fluctuations of the system which will have the greatest effect on the SFC.

As was done at the static sea-level operating point, a small perturbation in each input was made and its effect on the SFC, thrust and other engine conditions tabulated. The goal of these analyses is to develop a trim input, U_m, that improves the fuel economy at steady state without causing significant change in the operating conditions (both states and outputs).

The effects of the control inputs on several parameters are plotted on Figures IV-1 through IV-4. Figures IV-1 shows, again, the large effect nozzle area (u2) and fuel flow (u1) have on specific fuel consumption, the effect of inlet guide vanes (u3) is drastically reduced from that observed at static, sea level operation. Figure IV-2 indicates the offsetting effect of u1 and u2 and again indicates a zero net thrust control can be found. Fan speed (x1) and compressor

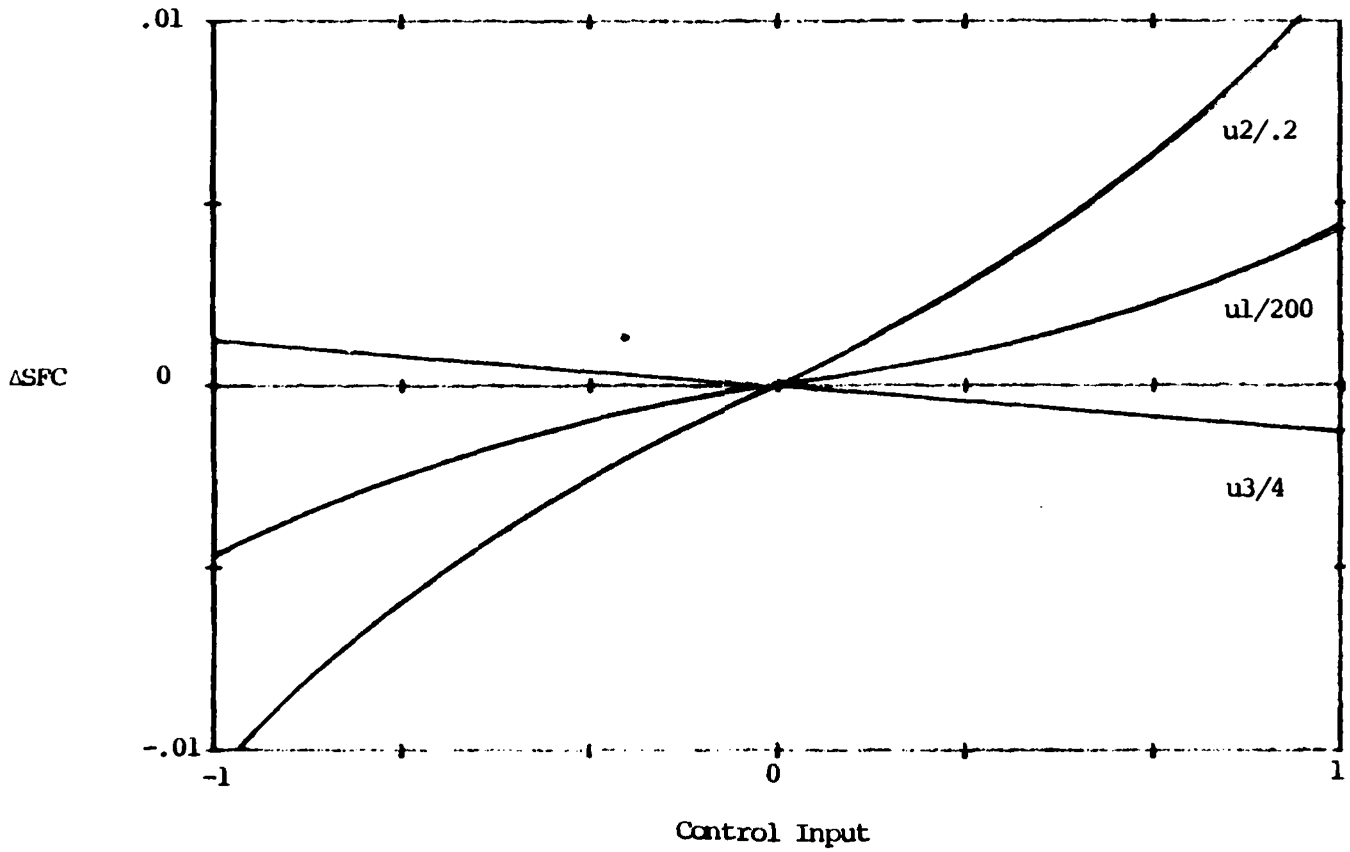


Figure IV-1. SFC vs Control Inputs

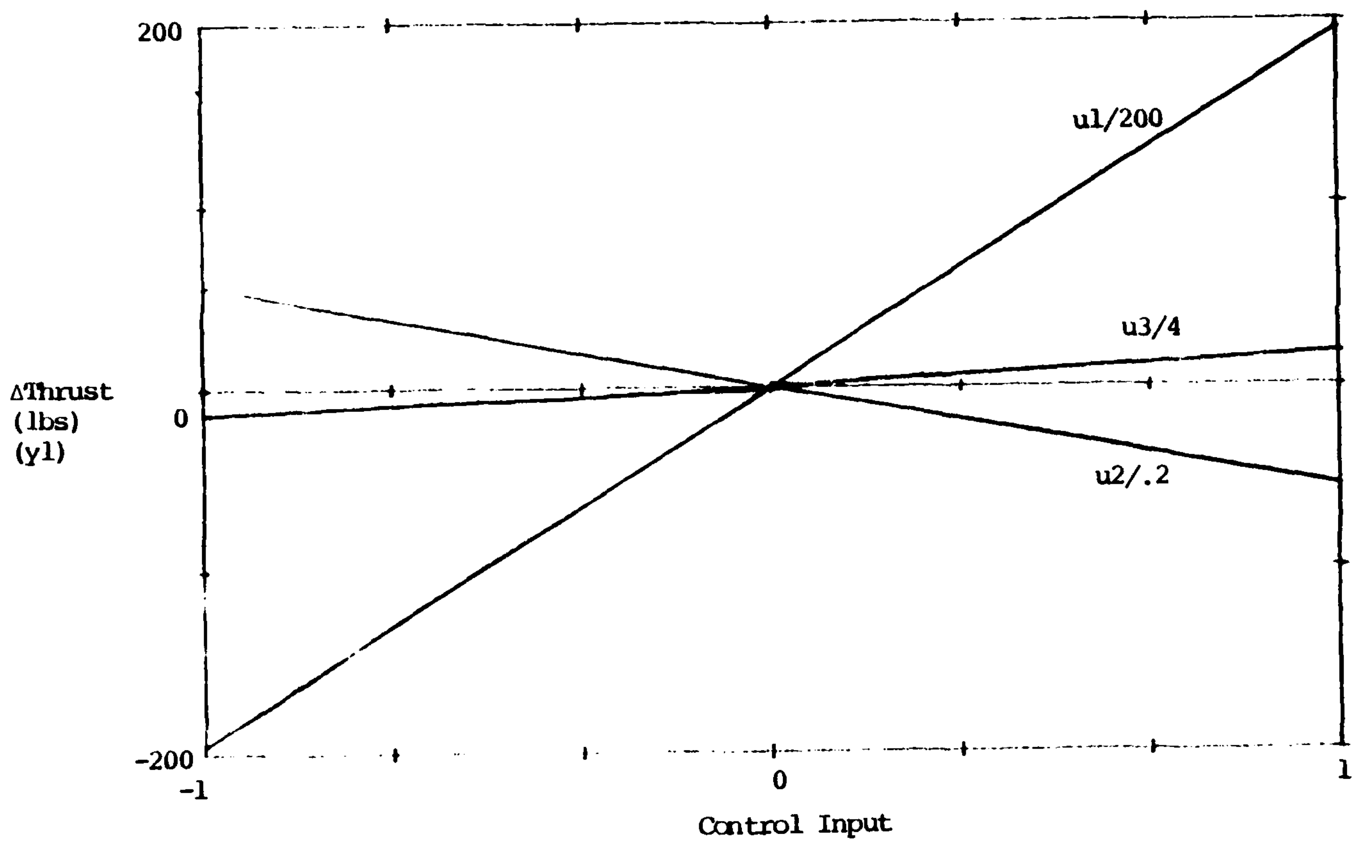


Figure IV-2. Thrust vs Control Inputs

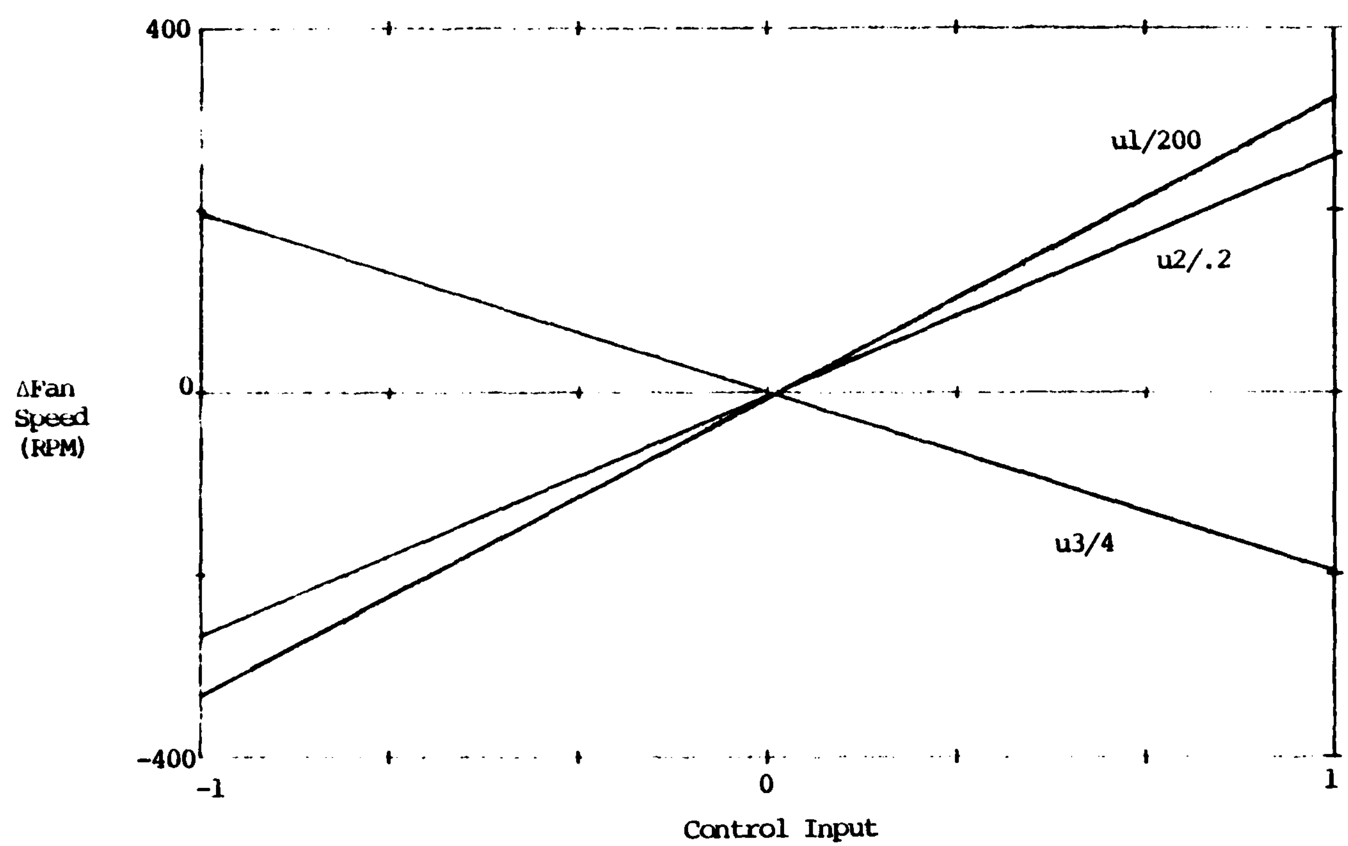


Figure IV-3. Fan Speed vs Control Inputs

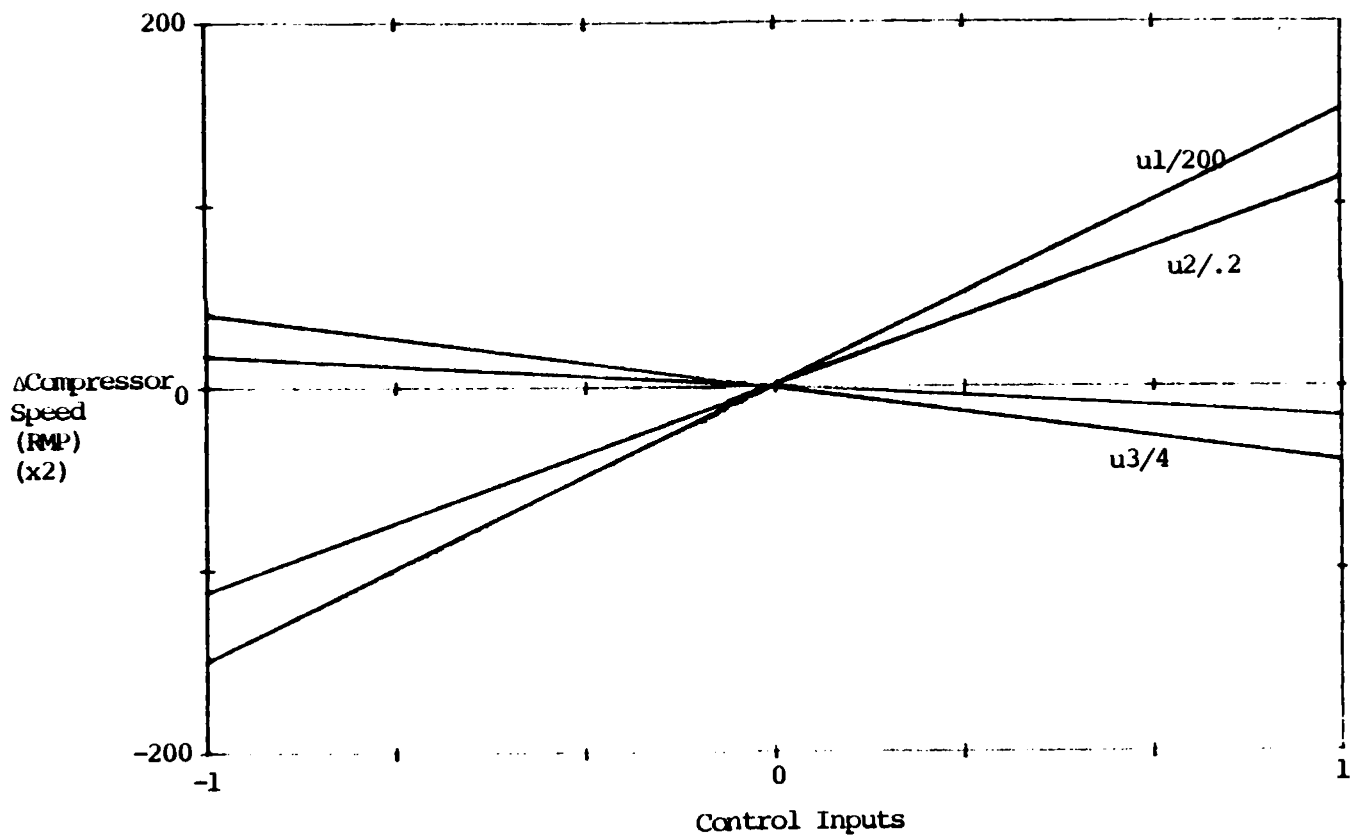


Figure IV-4. Compressor Speeds vs Control Inputs

speed (x2) effects are shown in Figures IV-3 and IV-4, respectively. At this operating point the inlet guide vane position (u3) may be most useful in controlling the fan speed fluctuation and thereby stabilizing the fan stability margin (y4). A sample control would close u2 by .2 square feet and reduce fuel flow by 57 pounds per hour. This combination produces a 1.4% decrease in specific fuel consumption and results in a 4% reduction in fan speed. A trim condition control input, U_m, would be

$$U_m = \begin{bmatrix} -57 \\ -.2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

B. REDUCED ORDER SIMULATION ANALYSIS

The model developed in Section III.C and Section III.D for the high altitude flight operating point was used to initially analyze the control law developed in Section IV.A. A reduced order controller is simpler and cheaper to implement through a digital computer. It requires less computer space and far less computing time than the full-order linear model developed controller.

A comparison of the three cases for which gain matrices were found indicated that Case 2 gave the most desirable results. The Case 2 weighting matrix was applied to the third order reduced system and perturbations applied to various

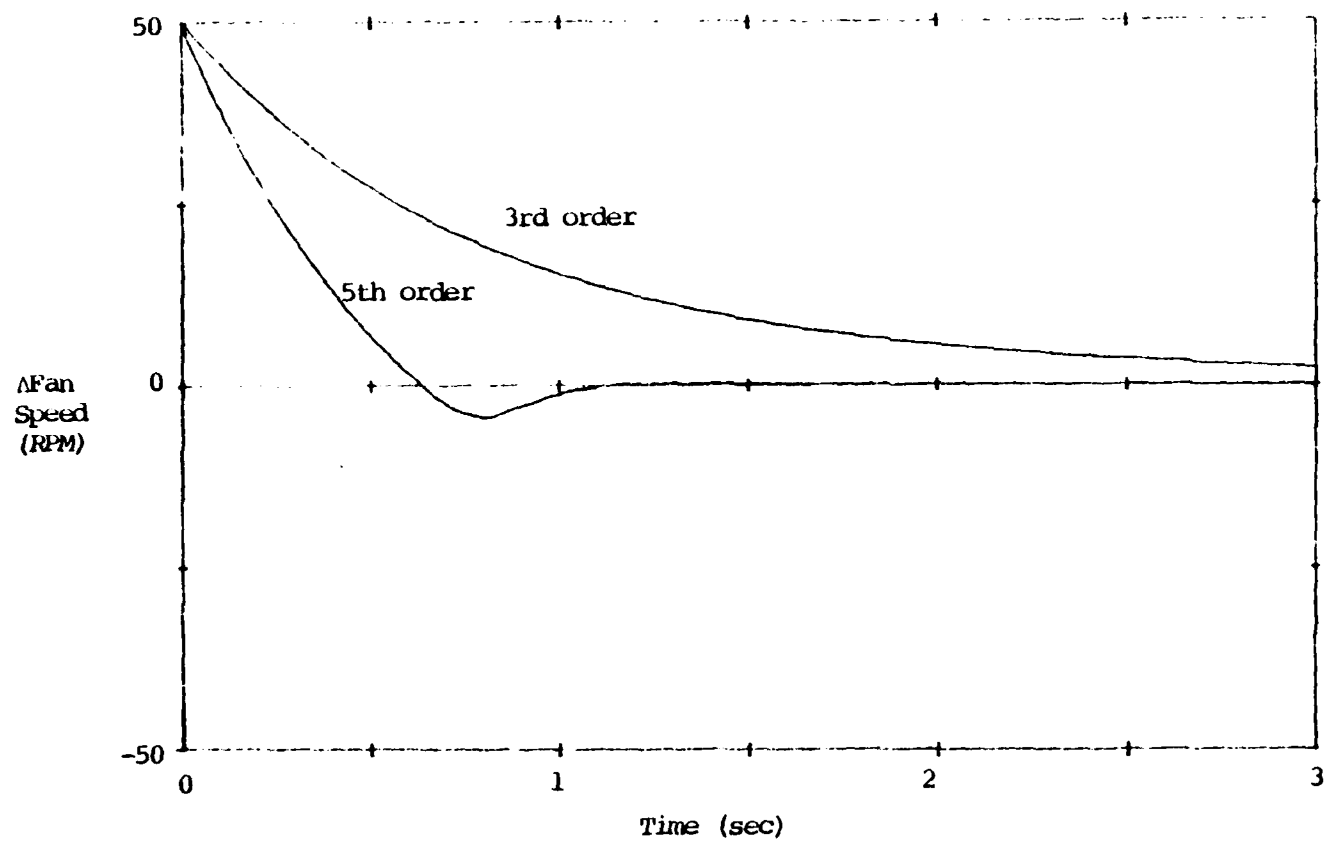


Figure IV-5. Third Order versus Fifth Order Control Response

states. The same weighting matrix was applied to the fifth order model (#3) and similar perturbations applied. Figure IV-5 shows the system response to a 50 RPM perturbation of the fan speed. The fifth order system has a settling time of approximately .45 seconds whereas the third order system has one of 1.5 seconds. This variation is accounted for by the two fast response poles in the fifth order model, (-51.65,j7.14).

These results must be compared to the 16th order system to determine which model most closely approximates the full order system response.

C. FULL ORDER SIMULATION ANALYSIS

The proof of the viability of a reduced order controller is its ability to control the full order system in the same manner as predicted in the reduced order model. The two reduced models analyzed in Section IV.B were applied to the sixteenth order linear model. Additionally, the Case 2 weighting matrix was applied and a full order controller was found and implemented. All simulation was done using the IBM Continuous System Modelling Program, Model III.

All three models were compared with similar perturbations applied. Figure IV-6 compares the full order and fifth order response to the 50 RPM speed perturbation. Figure IV-7 compares the full order and third order response to the same perturbation. Figure IV-8 compares the two reduced order controllers and the full order controller for the same fan speed perturbation.

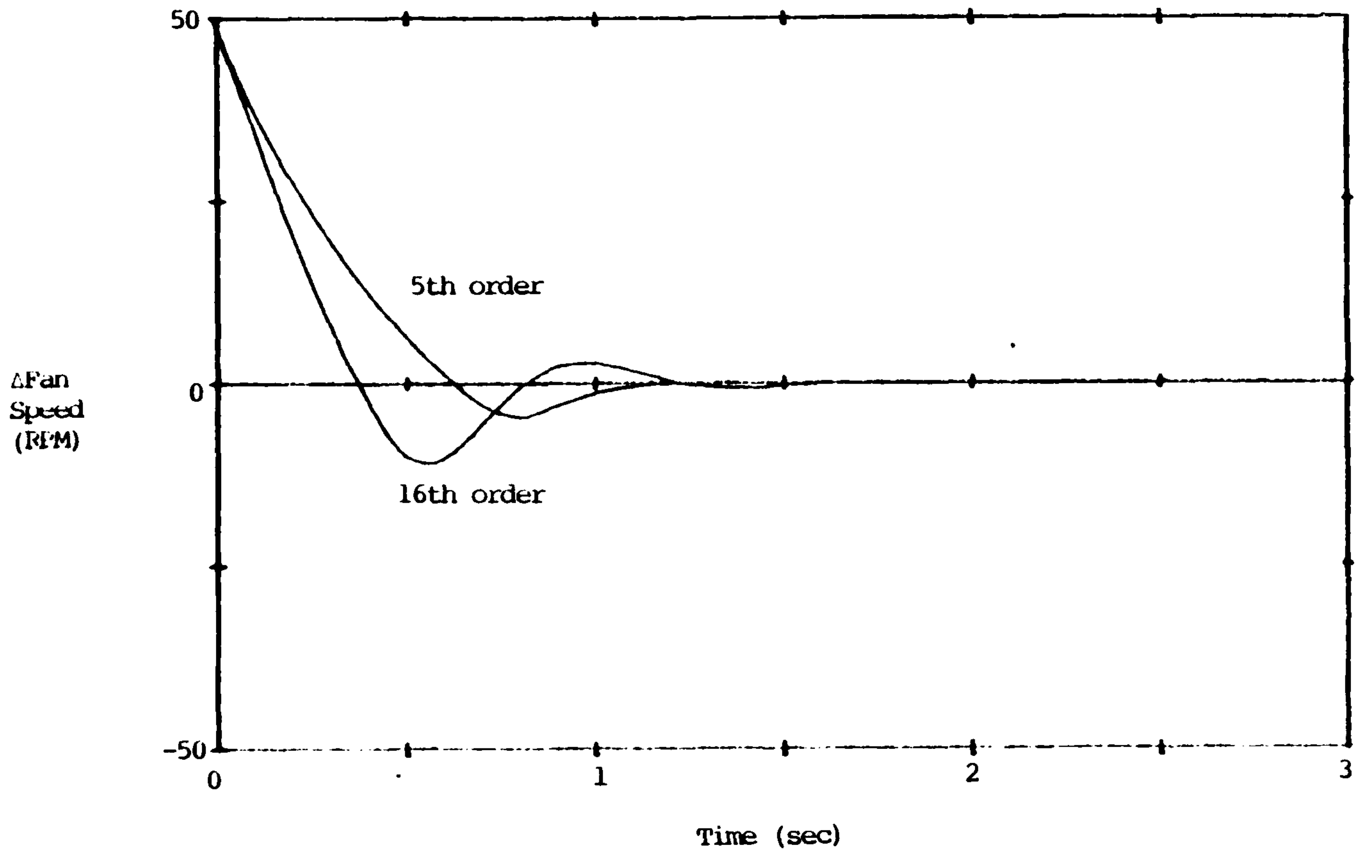


Figure IV-6. Fifth Order vs Full Order Control Response

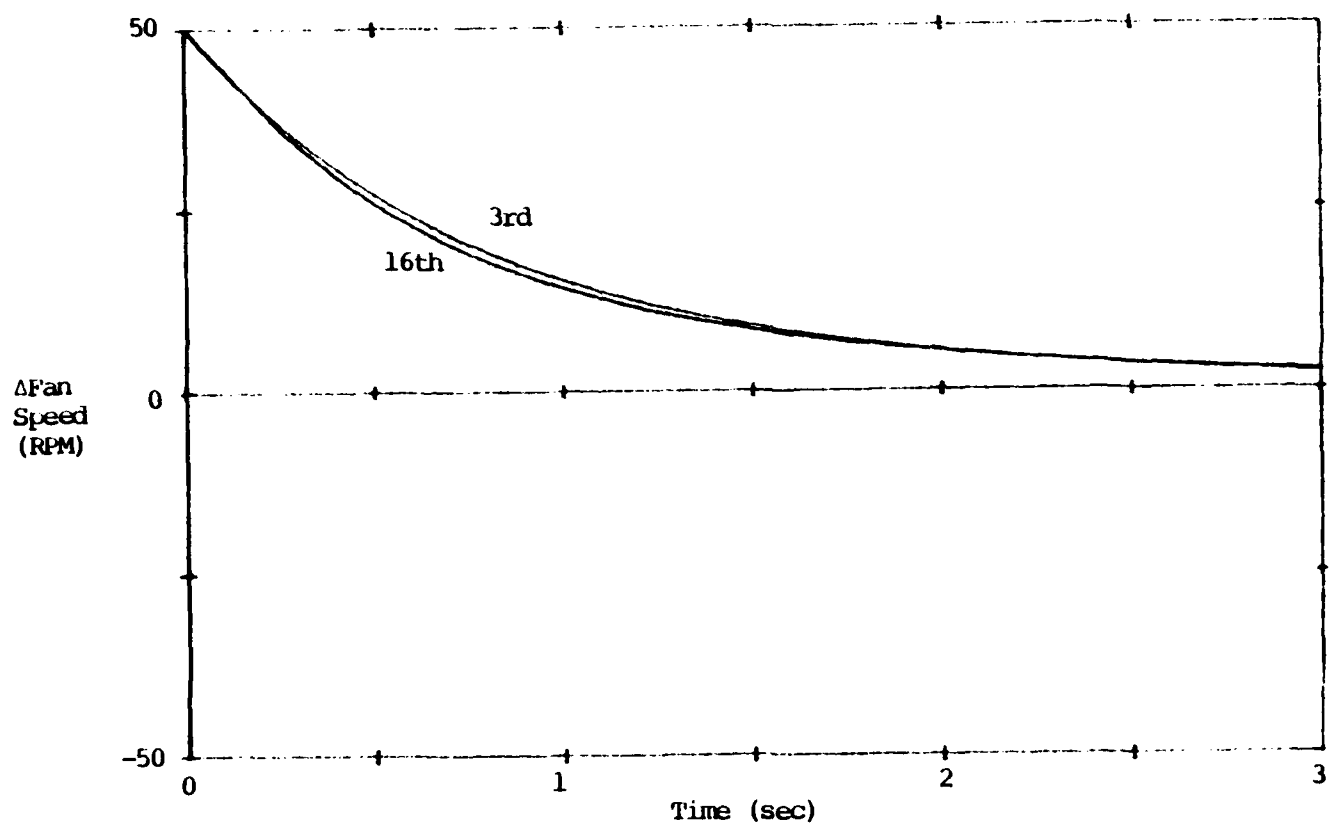


Figure IV-7. Third Order vs Full Order Control Response

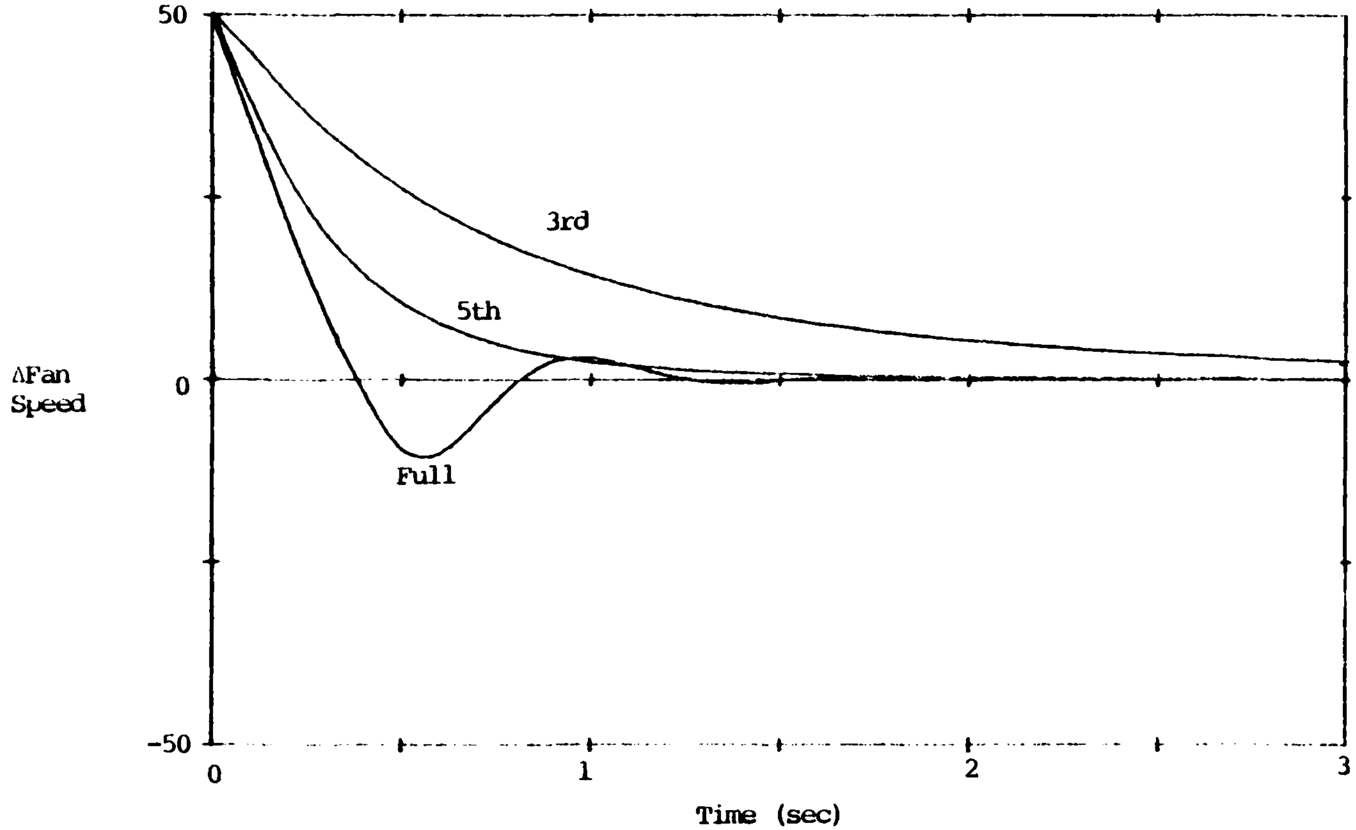


Figure IV-8. Full Order Implementation

The results show satisfactory agreement between the reduced order and full order implementation of the reduced order controller. This proves the viability of the controller and implies that comparable results will be found if the control is implemented in the non-linear dynamic system. The fifth order controller, with two poles outside the control bandwidth, most closely models the response of the full order controller. As was seen previously, the inclusion of the conjugate pair of poles dramatically improves the response time of the model and provides a much closer approximation to the full order system response.

This indicates that the fifth order model is sufficient for the control development process at this operating point. The two fast response poles outside the control bandwidth must be included to achieve this close simulation.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The study of implementing a reduced order controller that is flight regime dependent shows that at a specific operating point an improvement in specific fuel consumption can be made. The conclusions reached at the end of this research are:

1. The variable geometry existing in the state-of-the-art turbofan engines can be used to further improve the specific fuel consumption;

2. A reduced order controller can be found and implemented that concentrates control authority on the variable geometry rather than the fuel control, thus furthering the fuel savings; and

3. A similar controller can be found and implemented at any given operating point throughout the flight regime, these controllers could then be organized into a single digital machine to provide a regime-dependent engine control to improve the airframe/engine interaction in all flight conditions.

B. RECOMMENDATIONS FOR FUTURE STUDY

One obvious extension of this work is to find further reduced order controllers.

Additionally, work can be done to fully implement these reduced order controllers in the non-linear model using either the Pratt & Whitney transient simulation digital model or the hybrid computer model.

The effect on fatigue life and life cycle cost due to the "fuel saving" control schemes presented was not determined. It is therefore recommended that these factors be explored in order to fully analyze the usefulness of these control schemes.

REDUCR2 LISTING

```
C*****************************************************************************
C
C      THIS PROGRAM IS DESIGNED TO REDUCE A HIGH ORDER LINEAR MODEL
C      TO A REDUCED ORDER REPRESENTATIVE MODEL GIVEN THE INPUTS OF
C      DESIRED STATES AND ASSOCIATED EIGENVALUES.
C
C*****************************************************************************
      INTEGER*4 I,J,K,L,M,Q,NP,NC,NO,NR,IROW(8)
      REAL*8 AM(16,16),BM(16,8),CM(8,16),DM(8,8),T(16,16),DEL
     1,PA(8,8),PB(8,16),PC(16,8),PD(16,16),MA(16,16),MB(16,16)
     2,MC(16,16),GM(16,8),CTWO(8,16),BTWO(16,8),TFIN(16,16)
     3,FR(8,8),GR(8,8),HR(8,8),DR(8,8),HSTAR(8,16),FM(8,8)
     4,HONE(8,8),HTWO(8,16),DSTAR(16,8)
     5,DUM1(16,16),DUM3(8,16),DUM4(8,8),DUM2(16,8),CSTAR(8,8)
     6,DUM6(16,16),ZM(16,16),TM(16,16),AT(16,16),LM(16,16)
     7,RM(16,16),DUM5(8,16),DUM7(8,16)
      DATA PA/64*0.D0/,PB/128*0.D0/,PC/128*0.D0/,CSTAR/64*0.D0/
     1,PD/256*0.D0/,DUM1/256*0.D0/,DUM2/128*0.D0/,CTWO/128*0.D0/
     2,DUM3/128*0.D0/,DUM4/64*0.D0/,BTWC/128*0.D0/
     4,MA/256*0.D0/,MB/256*0.D0/,GM/128*0.D0/
     5,FR/64*0.D0/,GR/64*0.D0/,HR/64*0.D0/,FM/64*0.D0/
     6,DR/64*0.D0/,HSTAR/128*0.D0/,DUM5/128*0.D0/
     7,RM/256*0.D0/,DUM7/128*0.D0/
  980 FORMAT(6E12.5)
  981 FORMAT(8(I2,5X))
  982 FORMAT(6(1X,1PD12.5))
  983 FORMAT(4(I2,5X))
  984 FORMAT(6(5X,I3))
  985 FORMAT('0THE DESIRED REDUCED STATE ORDER IS ',I3)
  986 FORMAT('0THE EIGENVALUE DIAGONAL MATRIX IS ')
  989 FORMAT('0THE RIGHT EIGENVECTOR MATRIX IS ')
  990 FORMAT('0THE PLANT MATRIX IS ')
  992 FORMAT('0THE SIGNIFICANT STATES ARE ')
  993 FORMAT('0THE T11 MATRIX IS ')
  994 FORMAT('0THE T12 MATRIX IS ')
  995 FORMAT('0THE T21 MATRIX IS ')
  996 FORMAT('0THE T22 MATRIX IS ')
  997 FORMAT('0THE TRANSPOSE MATRIX IS ')
  998 FORMAT('0THE REDUCED PLANT MATRIX IS ')
 1001 FORMAT('0THE RM MATRIX IS ')
 1003 FORMAT('0THE REDUCED CONTROL DISTRIBUTION MATRIX IS ')
 1004 FORMAT('0THE REDUCED OUTPUT DISTRIBUTION MATRIX IS ')
 1005 FORMAT('0THE REDUCED FEEDFORWARD MATRIX IS ')
 1006 FORMAT(5(1X,E12.5))
 1007 FORMAT(' 2',/,I3)
```

```
C**********************************************************************
C
C     READ IN THE FULL ORDER MODEL MATRICES AND THE EIGENVECTOR MATRIX.
C
C**********************************************************************
      READ(5,983) NP,NC,NO,NR
      WRITE(6,990)
      DO 10 I=1,NP
   10 READ(5,980)(AM(I,J),J=1,NP)
      CALL RAPRNT(NP,NP,NP,6,AM,4,'(6(1X,1PD12.5))')
      READ(5,980)((CM(I,J),J=1,NP),I=1,NO)
      READ(5,980)((DM(I,J),J=1,NC),I=1,NO)
      READ(5,980)((BM(I,J),J=1,NC),I=1,NP)
      WRITE(6,989)
      DO 20 K=1,3
      L=K*6
      Q=L-5
      IF(Q.LT.13) GO TO 30
      Q=13
      L=16
   30 DC 20 I=1,NP
   20 READ(5,982)(T(I,J),J=Q,L)
      CALL RAPRNT(NP,NP,NP,6,T,4,'(6(1X,1PD12.5))')
C**********************************************************************
C
C     READ IN THE DESIRED REDUCED ORDER AND THE DESIRED STATES.
C     FCRM THE REORDERED STATE VECTOR X'
C
C             < X' > = < RM > * < X >
C
C**********************************************************************
      WRITE(6,985) NR
      READ(5,981)(IROW(I),I=1,NR)
      WRITE(6,992)
      WRITE(6,984)(IROW(I),I=1,NR)
      K=NR
      DC 42 I=1,NP
      DC 43 J=1,NR
   43 IF(I.EQ.IROW(J)) GO TO 44
      K=K+1
      RM(K,I)=1.D0
      GC TO 42
   44 RM(J,I)=1.D0
   42 CCNTINUE
      WRITE(6,1001)
      CALL RAPRNT(NP,NP,NP,6,RM,4,'(6(1X,1PD12.5))')
```

```
C*****************************************************************
C
C     CCMPUTATION OF THE DIAGONAL EIGENVALUE MATRIX.
C           <MB> = <T**-1> * <AM> * <T>
C
C*****************************************************************
      CALL MI(NP,NP,T)
      CALL MAMULT(NP,NP,NP,T,AM,MA,NP,NP,NP)
      CALL MI(NP,NP,T)
      CALL MAMULT(NP,NP,NP,MA,T,MB,NP,NP,NP)
      DO 90 I=1,NP
      DO 90 J=1,NP
      DEL=DABS(MB(I,J))
      IF(DEL.GT.(3.D-02)) GO TO 90
      MB(I,J)=0.D0
   90 CONTINUE
      CALL PRNT2(NP,NP,MB)
C*****************************************************************
C
C     DIVIDE THE EIGENVECTOR MATRIX INTO THE FOUR SUBMATRICES
C     USED IN THE ORDER REDUCTION.
C           <A'> = <RM>*<AM>*<RM**-1> = < PA | PB >
C                                       < PC | PD >
C
C*****************************************************************
      CALL MAMULT(16,16,16,RM,AM,DUM1,NP,NP,NP)
      CALL MI(NP,NP,RM)
      CALL MAMULT(16,16,16,DUM1,RM,TFIN,NP,NP,NP)
      CALL MI(NP,NP,RM)
      WRITE(6,997)
      CALL RAPRNT(NP,NP,NP,6,TFIN,4,'(6(1X,1PD12.5))')
      DO 200 I=1,NR
      DO 200 J=1,NR
  200 PA(I,J)=TFIN(I,J)
      WRITE(6,993)
      CALL RAPRNT(8,NR,NR,6,PA,4,'(6(1X,1PD12.5))')
      Q=NR+1
      M=NP-NR
      DO 210 I=1,NR
      DO 210 J=Q,NP
      P=J-NR
  210 PB(I,P)=TFIN(I,J)
      WRITE(6,994)
      CALL RAPRNT(8,NR,M,6,PB,4,'(6(1X,1PD12.5))')
      DO 220 I=Q,NP
      DO 220 J=1,NR
      P=I-NR
  220 PC(P,J)=TFIN(I,J)
```

```
      WRITE(6,995)
      CALL RAPRNT(16,M,NR,6,PC,4,'(6(1X,1PD12.5))')
      DC 230  I=Q,NP
      DC 230  J=Q,NP
  230 PD((I-NR),(J-NR))=TFIN(I,J)
      WRITE(6,996)
      CALL RAPRNT(16,M,M,6,PD,4,'(6(1X,1PD12.5))')
C**********************************************************************
C
C     CCMPUTE THE REDUCED PLANT MATRIX.
C           < FR > = < PA > - < PB >*< PD**-1 >*< PC >
C
C**********************************************************************
      CALL MI(16,M,PD)
      CALL MAMULT(16,16,8,PD,PC,DUM2,M,M,NR)
      CALL MAMULT(8,16,8,PB,DUM2,FM,NR,M,NR)
      DC 227  I=1,NR
      DC 227  J=1,NR
  227 FR(I,J)=PA(I,J)-FM(I,J)
      WRITE(6,998)
      CALL RAPRNT(8,NR,NR,6,FR,4,'(6(1X,1PD12.5))')
C**********************************************************************
C
C     COMPUTE THE REDUCED ORDER INPUT MATRIX.
C           | B1 |  = < RM >*< BM >
C           | B2 |
C           < GR > = < B1 > - < PB >*< PD**-1 >*< B2 >
C
C**********************************************************************
      CALL MAMULT(NP,NP,8,RM,BM,GM,NP,NP,NC)
C     WRITE(6,999)
C     CALL RAPRNT(NP,NP,NC,6,GM,4,'(6(1X,1PD12.5))')
      DO 250  I=1,NR
      DO 250  J=1,NC
  250 GR(I,J)=GM(I,J)
      DC 260  I=1,M
      DC 260  J=1,NC
  260 BTWO(I,J)=GM((I+NR),J)
      CALL MAMULT(8,16,16,PB,PD,DUM3,NR,M,M)
      CALL MAMULT(8,16,8,DUM3,BTWO,DUM4,NR,M,NC)
      DC 270  I=1,NR
      DO 270  J=1,NC
  270 GR(I,J)=GR(I,J)-DUM4(I,J)
      WRITE(6,1003)
      CALL RAPRNT(8,NR,NC,6,GR,4,'(6(1X,1PD12.5))')
```

```
C*************************************************************************
C
C      COMPUTE THE REDUCED ORDER OUTPUT MATRICES.
C             < C1 | C2 > = < CM >*< RM**-1 >
C             < HR > = < C1 > - < C2 >*< PD**-1 >*< PC >
C             < DR > = < DM > - < C2 >*< PD**-1 >*< B2 >
C
C*************************************************************************
      CALL MI(NP,NP,RM)
      CALL MAMULT(8,16,16,CM,RM,DUM5,NO,NP,NP)
      DO 300 I=1,NO
      DO 300 J=1,NR
  300 HR(I,J)=DUM5(I,J)
      DO 310 I=1,NO
      DO 310 J=1,M
  310 CTWO(I,J)=DUM5(I,(NR+J))
      CALL MAMULT(8,16,16,CTWO,PD,HSTAR,NO,M,M)
      CALL MAMULT(8,16,8,HSTAR,PC,CSTAR,NO,M,NR)
      DO 320 I=1,NO
      DO 320 J=1,NR
  320 HR(I,J)=HR(I,J)-CSTAR(I,J)
      CALL MAMULT(8,16,16,CTWO,PD,DUM7,NO,M,M)
      CALL MAMULT(8,16,8,DUM7,BTWO,DSTAR,NO,M,NC)
      DO 370 I=1,NO
      DO 370 J=1,NC
  370 DR(I,J)=DM(I,J)-DSTAR(I,J)
      WRITE(6,1004)
      CALL RAPRNT(8,NO,NR,6,HR,4,'(6(1X,1PD12.5))')
      WRITE(6,1005)
      CALL RAPRNT(8,NO,NC,6,DR,4,'(5(1X,1PD12.5))')
      WRITE(7,1006)((FR(I,J),J=1,NR),I=1,NR)
      WRITE(7,1006)((HR(I,J),J=1,NR),I=1,NO)
      WRITE(7,1006)((DR(I,J),J=1,NC),I=1,NO)
      WRITE(7,1006)((GR(I,J),J=1,NC),I=1,NR)
      WRITE(8,1007) NR
      DO 371 I=1,NR
  371 WRITE(8,980) (FR(I,J),J=1,NR)
      STOP
      END
      SUBROUTINE MI(M,N,AM)
C*************************************************************************
C
C  REFERENCE: FORTRAN COMPUTER PROGRAMS BY C. W. MERRIAM III
C             LEXINGTON BOOKS
C             1978, D. C. HEATH AND COMPANY
C             LEXINGTON, MASSACHUSETTS
C
C*************************************************************************
```

```
C
C     INVERSE OF AM BY THE PIVOT METHOD
C     REAL*8 AM(N,N),DM(N,N),DV(N),EV(N),D,E,DABS
C     INTEGER*4 FV(N),GV(N)
C     LOGICAL*1 HV(N)
C
C*************************************************************************
      REAL*8 AM(M,M)
      REAL*8 DM(30,30),DV(30),EV(30),D,E,DABS
      INTEGER*4 FV(30),GV(30)
      LOGICAL*1 HV(30)
1000  FORMAT('1')
1001  FORMAT(//,'     SINGULAR MATRIX')
      DO 1002 I=1,N
      HV(I)=.FALSE.
      DO 1002 J=1,N
1002  DM(I,J)=AM(I,J)
      DO 1008 K=1,N
      L=0
      D=0.D0
      DO 1003 J=1,N
      IF (HV(J)) GO TO 1003
      E=DABS(DM(K,J))
      IF (E .LE. D) GO TO 1003
      D=E
      L=J
1003  CONTINUE
      IF (L) 1004,1004,1005
1004  WRITE (6,1001)
      WRITE (6,1000)
      RETURN
1005  D=1.D0/DM(K,L)
      DO 1006 I=1,N
      DV(I)=D*DM(I,L)
      EV(I)=DM(K,I)
      DM(I,L)=0.D0
1006  DM(K,I)=0.D0
      DV(K)=D
      EV(L)=-1.D0
      DO 1007 I=1,N
      DO 1007 J=1,N
1007  DM(I,J)=DM(I,J)-DV(I)*EV(J)
      DM(K,L)=D
      FV(K)=L
      GV(L)=K
1008  HV(L)=.TRUE.
```

```
      DO 1009 I=1,N
      K=FV(I)
      DO 1009 J=1,N
      L=GV(J)
 1009 AM(K,L)=DM(I,J)
      RETURN
      END
      SUBROUTINE RAPRNT(NMAX,M,N,L,A,IDIM,FMT)
C************************************************************************
C
C     SUBROUTINE RAPRNT ALLOWS VARIABLE FORMAT OUTPUT.
C     IT IS MOST USEFUL IN PRINTING ARRAYS IN COLUMN FORMATS.
C     THIS SUBROUTINE TECHNIQUE WAS DEVELOPED FROM THAT USED IN
C     OPTSYS4. (STANFORD UNIVERSITY)
C
C************************************************************************
      REAL*8 A(NMAX,N)
      DIMENSION FMT(IDIM)
      NU=L
      DO 20 NL=1,N,L
      IF (NU.GT.N) NU=N
      DO 10 I=1,M
   10 WRITE(6,FMT)(A(I,J),J=NL,NU)
      WRITE(6,100)
  100 FORMAT(' ')
   20 NU=NU+L
      RETURN
      END
      SUBROUTINE MAMULT(L,M,N,F,G,H,NL,NM,NN)
C************************************************************************
C
C     SUBROUTINE MAMULT PRODUCES THE MATRIX PRODUCT OF THE TWO
C     INPUT MATRICES, F ANF G, AND RETURNS THAT RESULT AS H.
C
C************************************************************************
      INTEGER*4 I,J,K
      REAL*8 F(L,M),G(M,N),H(L,N),DEL
      DO 1000 I=1,NL
      DO 1000 K=1,NN
      DEL=0.D0
      DO 1010 J=1,NM
 1010 DEL=DEL+F(I,J)*G(J,K)
 1000 H(I,K)=DEL
      RETURN
      END
```

```
      SUBROUTINE PRNT2(NMAX,N,A)
C*****************************************************************************
C
C     SUBROUTINE PRNT2 IS USED TO PRINT THE EIGENVALUES OF THE
C     DIAGONAL EIGENVALUE MATRIX, A, IN COMPLEX FORM.
C
C*****************************************************************************
      INTEGER*4 I,J,M
      REAL*8 A(NMAX,NMAX),B(32),D
      M=2*N
      DO 5 I=1,M
    5 B(I)=0.D0
  100 FORMAT(' EIGENVALUE # ',I2,' IS ',E12.5,' +J( ',E12.5,' )')
      DO 10 I=1,N
      J=N+I
      IF(I.EQ.1) GO TO 20
      D=A((I-1),I)
      IF(D.EQ.(0.D0)) GO TO 20
      B(J)=D
   20 IF(I.EQ.N) GO TO 10
      D=A((I+1),I)
      IF(B(J).NE.(0.D0)) GO TO 10
      B(J)=D
   10 B(I)=A(I,I)
      DO 30 I=1,N
      J=I+N
   30 WRITE(6,100) I,B(I),B(J)
      RETURN
      END
```

REDUCR2 SAMPLE INPUT

```
16         5          7          5
   -.2201E+01  .2888E+00   .4578E+01   .4070E+03  -.7133E+03  -.3902E+01
   -.5671E+00 -.4913E+00   .5303E+01   .6569E+01  -.1989E+01   .4717E+01
    .4372E+01 -.3571E+00  -.1355E+00   .3669E-01
   -.2508E+00 -.3299E+01   .1254E+03  -.2448E+03  -.2891E+03   .1602E+02
   -.1113E+01 -.1466E+01   .5483E+01   .4444E+01  -.2090E+01  -.1690E+00
   -.1547E+01 -.1970E+00  -.6135E+00  -.2487E+00
    .6325E+00  .3280E+01  -.1451E+03  -.2059E+02   .7103E+03  -.3635E+02
    .3108E+01  .3127E+01   .4272E+00   .3922E+01   .2956E+01   .5757E+00
    .5284E+01  .6465E+00   .1919E+01   .8275E+00
    .2170E+00 -.6866E-02   .1222E+03  -.5477E+03   .5056E+02  -.4546E+01
   -.3733E+00 -.1261E+01  -.1548E+01  -.2937E+01   .2592E+01  -.2320E+00
   -.2062E+01 -.2401E+00  -.5207E+00  -.3302E+00
    .2308E-02 -.8184E-02   .1030E-01   .1441E+01  -.9184E+01   .7554E-01
   -.2174E-01 -.1385E-01  -.1009E-02  -.4387E-01  -.2164E-01  -.7064E-02
   -.6398E-01  .2193E-01   .4212E-02  -.1019E-01
    .9654E+00 -.6180E-01   .1344E+01   .2993E+01   .7466E+02  -.1881E+02
    .4322E+00  .1085E+00   .6355E-01   .5642E+00   .5961E-01   .8477E-01
    .7630E+00  .9544E-01   .2768E+00   .1190E+00
    .6313E+00 -.1241E+00   .1587E+01   .8980E+01   .1286E+03   .2376E+01
   -.1943E+02  .1005E+00   .5668E-01   .5217E+00   .5703E-01   .7446E-01
    .7114E+00  .8928E-01   .3296E+00   .1101E+00
   -.1584E+00  .1046E+01   .3124E+02  -.7562E+01  -.1695E+03   .2622E+02
   -.8973E+00 -.2026E+02  -.1537E-01  -.1403E+00  -.1380E+00  -.2051E+00
   -.1846E+01 -.2284E+00  -.6277E+00  -.2960E+00
   -.1000E-01 -.1878E-01   .5945E-01  -.4501E+00  -.1170E+02   .5441E+00
   -.7179E-01  .6623E+01  -.5001E+02  -.1159E+00   .3936E+02  -.1718E-01
   -.1418E+00 -.1539E-01  -.3923E-01  -.2446E-01
   -.1505E-03 -.2800E-03   .7927E-03  -.6752E-02  -.1739E+00   .8019E-02
   -.1101E-02  .9811E-01  -.6668E+00  -.6685E+00   .5830E+00  -.2673E-03
   -.2234E-02 -.2463E-03  -.6182E-03  -.3805E-03
   -.1849E+01 -.8250E+01   .1237E+02  -.8911E+02  -.1984E+04   .9616E+02
   -.1046E+02  .3414E+02  -.1780E+01  -.1617E+02  -.4932E+02  -.2386E+01
   -.2172E+02 -.2635E+01  -.7244E+01  -.3412E+01
   -.2437E-01 -.4384E+00  -.1125E+03   .4171E+03  -.2851E+02   .1554E+01
   -.1256E+00  .9820E+01  -.9810E+01  -.9374E+01   .3856E+02  -.5002E+02
   -.2191E+00 -.6157E-02  -.6063E-01  -.1019E-01
   -.1087E-02 -.1948E-01  -.5002E+01   .1854E+02  -.1269E+01   .6902E-01
   -.5599E-02  .4364E+00  -.4360E+00  -.4390E+00   .1714E+01  -.2001E+01
   -.2010E+01 -.3079E-03  -.2711E-02  -.4892E-03
   -.1618E+00 -.2496E+00  -.5151E+02   .4204E+02   .3301E+03   .1353E+01
   -.8471E-01  .4527E+01  -.4511E+01  -.4498E+01   .1775E+02  -.3199E+01
   -.3253E+01 -.1974E+02  -.3424E-01   .1631E-02
   -.4822E-02  .2779E-01  -.1189E-01   .2126E+00   .6724E+01  -.3364E+00
    .1991E+02  .1173E+00   .7729E-02   .5024E-01   .2281E-01   .1031E-01
    .6702E-01  .1170E-01  -.1997E+02   .1060E-01
```

-.1052E+01	.1968E+01	.7313E+01	.2703E+03	.4611E+03	-.4110E+02
.3105E+01	.5921E+00	.1231E+01	.6251E+01	-.1854E+01	.8306E+00
.7565E+01	.2944E+02	.2467E+02	-.4380E+02		
.3693E-01	-.4391E+00	.5175E+01	.1056E+03	.1769E+02	.5681E+01
-.2791E+00	-.2998E+00	.2669E+00	.2226E+00	-.1059E+01	-.7790E-02
-.8248E-01	.1862E+01	.1378E+01	-.9131E-02	.6468E-02	.5301E-05
-.1301E-03	-.4923E-03	-.7060E-02	-.9845E-04	-.3141E-04	-.1604E-04
-.9393E-05	-.8453E-04	-.8049E-05	-.1253E-04	-.1128E-03	-.1347E-04
-.3121E-04	-.1783E-04	.0000E+00	.0000E+00	.0000E+00	.0000E+00
.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
.1000E+01	.0000E+00	.0000E+00	.0000E+00	.0000E+00	.0000E+00
.4451E-04	.2190E-04	-.4438E-04	-.1139E-02	-.2581E-01	-.4171E-03
-.1352E-03	-.3997E-04	-.2307E-04	-.2098E-03	-.2088E-04	-.3094E-04
-.2815E-03	-.3421E-04	-.9397E-04	-.4426E-04	.2815E-04	.2691E-03
-.5550E-02	.1340E-02	.3019E-01	-.2332E-02	.1587E-03	.4691E-04
.2701E-04	.2458E-03	.2448E-04	.3623E-04	.3299E-03	.4005E-04
.1103E-03	.5182E-04	.7026E-05	-.3220E-04	-.3100E-03	-.1545E-02
-.3494E-01	.2362E-03	.5487E-04	-.5397E-04	-.3119E-04	-.2842E-03
-.2821E-04	-.4221E-04	-.3837E-03	-.4643E-04	-.1266E-03	-.5996E-04
.7236E-05	.5656E-05	-.1492E-03	-.4182E-03	-.9378E-02	-.1243E-04
.4169E-04	-.1459E-04	-.8365E-05	-.7575E-04	-.7608E-05	-.1136E-04
-.1035E-03	-.1253E-04	-.3448E-04	-.1519E-04		
.3418E-01	-.3614E+02	.5643E+01	-.3991E+01	-.9300E+02	-.1121E-03
-.2275E+00	.3113E+00	.4912E-02	-.1314E+02	.0000E+00	.0000E+00
.0000E+00	.0000E+00	.0000E+00	.7793E-04	-.6176E-02	-.3654E-02
.5683E-06	-.4671E+00	-.8364E-04	.7969E-02	.1033E-02	-.2745E-02
.2704E+00	.1091E-03	.6103E-02	-.5733E-03	-.6403E-03	-.4560E+00
.2857E-04	.1150E-02	.3299E-03	-.3642E-04	-.1298E+00	
.2157E-01	-.2235E+03	-.7410E+02	.2375E+01	-.2774E+03	.5678E+00
-.1582E+02	-.8139E+01	-.2707E+02	.6483E+03	-.1516E+01	.6569E+02
.2203E+02	.3291E+02	-.2146E+05	.6232E+00	.1142E+03	-.2642E+01
.4141E+00	-.1414E+04	.1868E-01	-.3798E+02	.1151E+00	-.1367E+00
-.7513E+02	-.2184E+00	.3366E+02	.2102E+02	-.8417E-01	.1553E+04
-.2310E+00	.8260E+02	.2530E+02	-.5327E-01	.1991E+04	.5137E+00
-.1354E+02	-.6101E+01	.3687E+01	-.1942E+04	.1349E+00	.3215E+01
-.4624E+00	-.5426E+00	-.6172E+04	.2029E-02	.5796E-01	-.7080E-02
-.8187E-02	-.9146E+02	.2097E+02	.1313E+03	-.7298E+02	-.9602E+02
.5002E+05	.2956E+00	-.1615E+01	-.3980E+00	-.1226E+01	-.6791E+04
.1316E-01	-.7040E-01	-.4024E-01	-.5461E-01	-.3018E+03	.1503E+00
.5963E+01	.1501E+00	-.9370E+00	-.2008E+04	-.2450E-01	-.8056E+00
-.2742E+00	.3477E+00	.1193E+03	-.2152E+01	-.8843E+02	-.4743E+02
.2444E+02	.1242E+05				

4.49083D-01	5.02385D-01	-1.70463D-01	-6.45942D-03	-7.52346D-02	4.40652D-02
-2.62785D-01	1.89552D-01	-9.79510D-02	5.30588D-02	-2.35255D-02	1.30648D-01
-3.34017D-02	-5.81348D-01	1.16259D-02	-2.64895D-03	-1.18395D-03	-8.91677D-04
-5.93598D-01	-1.82331D-01	1.39966D-03	2.30708D-03	-1.05593D-03	1.41550D-03
1.62044D-03	1.90011D-03	1.26119D-04	1.49944D-06	-2.72370D-05	-4.33304D-05
2.21726D-03	4.60750D-03	-1.67715D-03	2.45223D-03	1.65117D-03	-8.05221D-04
9.18604D-03	1.42006D-02	-3.11352D-03	9.85780D-04	1.17462D-03	-6.54912D-05
-5.09709D-03	1.25003D-01	3.60642D-03	-4.19825D-03	2.18221D-04	1.07724D-03
7.32206D-03	1.08506D-02	5.89772D-01	-4.86408D-01	1.80846D-01	-8.83379D-01
1.07519D-04	1.55898D-04	7.85561D-03	-6.82040D-03	2.46195D-03	-1.17351D-02
-9.61172D-02	-5.05024D-02	1.77481D-01	2.06103D-01	-1.89722D-02	-1.65855D-01
5.06815D-01	1.09254D-01	1.00000D+00	0.0	6.87620D-01	4.08375D-01
2.24021D-02	4.75680D-03	4.18727D-02	-7.56130D-04	2.88031D-02	1.68113D-02
5.04231D-02	-1.61235D-01	1.68770D-01	-1.24504D-01	9.97470D-02	9.26778D-03
-1.37238D-04	-2.09382D-03	1.32145D-03	1.40829D-04	-7.66785D-04	1.08260D-04
3.24032D-01	5.32906D-01	-3.19013D-01	-4.89239D-01	-6.90730D-01	7.64297D-02
-1.97027D-01	-3.58500D-03	-2.00990D-01	-2.93615D-02	8.57289D-01	-9.02132D-01
-1.00450D-01	1.01558D-02	-1.23576D-01	-1.62488D-01	-1.62399D-01	3.39264D-01
3.53549D-02	1.31408D-02	1.54478D-02	1.10806D-03	1.01947D-02	9.73011D-03
6.55427D-03	1.85608D-03	2.90773D-03	-1.75116D-03	1.10960D-03	2.35310D-03
3.99519D-04	-1.06646D-03	3.12522D-04	1.10439D-04	6.16728D-03	-7.63737D-04
-4.77258D-02	-3.26159D-02	-1.36740D-02	7.36619D-02	1.32837D-01	-6.67265D-02
-8.44073D-02	8.44531D-03	-1.73616D-03	9.54052D-02	1.52706D-01	-5.64510D-02
3.65309D-01	7.42899D-02	2.47079D-01	1.91783D-01	1.56344D-01	-3.98913D-02
1.93810D-01	-1.05861D-01	4.01683D-01	2.54645D-01	3.91698D-02	-1.08466D-01
2.30557D-03	-1.10135D-03	4.77520D-03	2.59443D-03	2.09693D-04	7.57053D-04
7.51256D-02	-1.13510D-01	2.42124D-01	1.97667D-01	1.70613D-02	-1.29823D-01
1.18353D-01	-9.41748D-02	2.64286D-01	1.76348D-01	2.73188D-02	-9.27220D-02
4.49831D-03	-3.32275D-03	1.00577D-02	5.95856D-03	5.46183D-04	3.60426D-03
-9.65587D-02	9.22555D-01	7.20239D-02	5.07472D-01	1.90620D-01	-1.38228D-01
6.15255D-01	-3.23849D-02	-5.77231D-02	3.49762D-01	2.42231D-01	-6.75486D-02
6.01533D-01	1.00000D+00	0.0	6.37576D-01	2.74857D-01	-5.55243D-02
-7.94500D-01	-8.20671D-01	-5.30038D-02	3.93876D-01		
-5.92096D-01	-5.57166D-01	-5.76613D-02	8.93897D-01		
-8.99539D-03	-8.51248D-03	3.74201D-04	1.97724D-02		
-1.31871D-03	-1.53329D-03	-7.25229D-04	4.65406D-03		
3.54510D-04	-4.62904D-05	-7.14630D-04	1.52329D-03		
-4.32254D-02	-4.69223D-02	1.00564D-02	2.28251D-02		
-2.93135D-02	-3.43887D-02	-1.59676D-02	2.09551D-02		
-9.73261D-02	-9.79876D-02	7.51593D-02	1.12480D-01		
-1.87588D-02	-2.67022D-02	1.33973D-01	-2.32599D-02		
-6.08708D-02	2.25138D-03	1.50706D-03	7.79925D-04		
-7.18483D-03	-1.65207D-02	9.42613D-02	-4.71189D-02		
4.48151D-03	-1.72681D-02	8.80749D-02	-2.40229D-02		
6.61978D-04	-4.07244D-03	3.20394D-03	-1.59628D-01		
2.72455D-02	7.18832D-04	7.74621D-01	-1.17182D-02		
-3.14211D-02	-3.72423D-02	-2.20941D-01	2.47640D-02		
-3.76396D-03	-1.76218D-02	5.51728D-01	4.70281D-02		

1 2 5 11 16

REDUCR2 SAMPLE OUTPUT

```
THE PLANT MATRIX IS
 -2.201000D+00  2.888000D-01  4.578000D+00  4.070000D+02 -7.133000D+02 -3.902000D+00
 -2.508000D-01 -3.299000D+00  1.254000D+02 -2.448000D+02 -2.891000D+02  1.602000D+01
  6.325000D-01  3.280000D+00 -1.451000D+02 -2.059000D+01  7.103000D+02 -3.635000D+01
  2.170000D-01 -6.866000D-03  1.222000D+02 -5.477000D+02  5.056000D+01 -4.546000D+00
  2.308000D-03 -8.184000D-03  1.030000D-02  1.441000D+00 -9.184000D+00  7.554000D-02
  9.654000D-01 -6.180000D-02  1.344000D+00  2.993000D+00  7.466000D+01 -1.881000D+01
  6.313000D-01 -1.241000D-01  1.587000D+00  8.980000D+00  1.286000D+02  2.376000D+00
 -1.584000D-01  1.046000D+00  3.124000D+01 -7.562000D+00 -1.695000D+02  2.622000D+01
 -1.000000D-02 -1.878000D-02  5.945000D-02 -4.501000D-01 -1.170000D+01  5.441000D-01
 -1.505000D-04 -2.800000D-04  7.927000D-04 -6.752000D-03 -1.739000D-01  8.019000D-03
 -1.849000D+00 -8.250000D+00  1.237000D+01 -3.811000D+01 -1.984000D+03  9.616000D+01
 -2.437000D-02 -4.384000D-01 -1.125000D+02  4.171000D+02 -2.851000D+01  1.554000D+00
 -1.087000D-03 -1.948000D-02 -5.002000D+00  1.854000D+01 -1.269000D+00  6.902000D-02
 -1.618000D-01 -2.496000D-01 -5.151000D+01  4.204000D+01  3.301000D+02  1.353000D+00
 -4.822000D-03  2.779000D-02 -1.189000D-02  2.026000D-01  6.724000D+00 -3.064000D-01
 -1.052000D+00  1.968000D+00  7.313000D+00  2.703000D+02  4.611000D+02 -2.110000D+01

 -5.671000D-01 -4.913000D-01  5.303000D+00  6.569000D+00 -1.989000D+00  4.717000D+00
 -1.113000D+00 -1.466000D+00  5.483000D+00  4.444000D+00 -2.090000D+00 -1.690000D-01
  3.108000D+00  3.127000D+00  4.272000D-01  3.922000D+00  2.956000D+00  5.757000D-01
 -3.733000D-01 -1.261000D+00 -1.548000D+00 -2.937000D+00  2.592000D+00 -2.320000D-01
 -2.174000D-02 -1.385000D-02 -1.009000D-03 -4.387000D-02 -2.164000D-02 -7.064000D-03
  4.322000D-01  1.085000D-01  6.355000D-02  5.642000D-01  5.961000D-02  8.477000D-02
 -1.943000D+01  1.005000D-01  5.668000D-02  5.217000D-01  5.703000D-02  7.446000D-02
 -8.973000D-01 -2.026000D+01 -1.537000D-02 -1.403000D-01 -1.380000D-01 -2.051000D-01
 -7.179000D-02  6.623000D+00 -5.001000D+01 -1.159000D-01  3.936000D+01 -1.718000D-02
 -1.101000D-03  9.811000D-02 -6.668000D-01 -6.685000D-01  5.830000D-01 -2.673000D-04
 -1.046000D+01  3.414000D+01 -1.780000D+00 -1.617000D+01 -4.932000D+01 -2.386000D+00
 -1.256000D-01  9.820000D+00 -9.810000D+00 -9.874000D+00  3.856000D+01 -5.002000D+01
 -5.599000D-03  4.364000D-01 -4.360000D-01 -4.390000D-01  1.714000D+00 -2.001000D+00
 -8.471000D-02  4.527000D+00 -4.511000D+00 -4.498000D+00  1.775000D+01 -3.199000D+00
  1.991000D+01  1.173000D-01  7.729000D-03  5.024000D-02  2.281000D-02  1.031000D-02
  3.105000D+00  5.921000D-01  1.231000D+00  6.251000D+00 -1.854000D+00  8.306000D-01

  4.872000D+00 -3.571000D-01 -1.355000D-01  3.669000D-02
 -1.547000D+00 -1.970000D-01 -6.135000D-01 -2.487000D-01
  5.234000D+00  6.465000D-01  1.919000D+00  8.275000D-01
 -2.062000D+00 -2.401000D-01 -5.207000D-01 -3.302000D-01
 -6.398000D-02  2.193000D-02  4.212000D-03 -1.019000D-02
  7.630000D-01  9.544000D-02  2.768000D-01  1.190000D-01
  7.114000D-01  8.928000D-02  3.296000D-01  1.101000D-01
 -1.846000D+00 -2.284000D-01 -6.277000D-01 -2.960000D-01
 -1.418000D-01 -1.539000D-02 -3.923000D-02 -2.446000D-02
 -2.234000D-03 -2.463000D-04 -6.182000D-04 -3.805000D-04
 -2.172000D+01 -2.635000D+00 -7.244000D+00 -3.412000D+00
 -2.191000D-01 -6.157000D-03 -6.063000D-02 -1.019000D-02
 -2.010000D+00 -3.079000D-04 -2.711000D-03 -4.892000D-04
```

-3.25300D+00	-1.97400D+01	-3.42400D-02	1.63100D-03
6.71200D-02	1.17000D-02	-1.99700D+01	1.06000D-02
7.56500D+00	2.94400D+01	2.46700D+01	-4.88000D+01

THE RIGHT EIGENVECTOR MATRIX IS

4.48083D-01	5.02385D-01	-1.70463D-01	-6.45942D-03	-7.52346D-02	4.40652D-02
-2.62785D-01	1.89552D-01	-9.79510D-02	5.30588D-02	-2.35255D-02	1.30648D-01
-3.34017D-02	-5.81348D-01	1.16259D-02	-2.64895D-03	-1.18395D-03	-8.91677D-04
-5.95598D-01	-1.82331D-01	1.39966D-03	2.30708D-03	-1.05593D-03	1.41550D-03
1.62044D-03	1.90011D-03	1.26119D-04	1.49944D-06	-2.72370D-05	-4.33304D-05
2.21726D-03	4.60750D-03	-1.67715D-03	2.45223D-03	1.65117D-03	-8.05221D-04
9.18604D-03	1.42006D-02	-3.11352D-03	9.85780D-04	1.17462D-03	-6.54912D-05
-5.09709D-03	1.25003D-01	3.60642D-03	-4.19825D-03	2.18221D-04	1.07724D-03
7.32206D-03	1.08506D-02	5.89772D-01	-4.86408D-01	1.80846D-01	-8.83379D-01
1.07519D-04	1.55898D-04	7.85561D-03	-6.82040D-03	2.46195D-03	-1.17351D-02
-9.61172D-02	-5.05024D-02	1.77481D-01	2.06103D-01	-1.89722D-02	-1.65855D-01
5.06815D-01	1.09254D-01	1.00000D+00	0.0	6.87620D-01	4.08375D-01
2.24021D-02	4.75680D-03	4.18727D-02	-7.56130D-04	2.88031D-02	1.68113D-02
5.04231D-02	-1.61235D-01	1.68770D-01	-1.24504D-01	9.97470D-02	9.26778D-03
-1.37238D-04	-2.09382D-03	1.32145D-03	1.40829D-04	-7.66785D-04	1.08260D-04
3.24032D-01	5.32906D-01	-3.19013D-01	-4.89239D-01	-6.90730D-01	7.64297D-02

-1.97027D-01	-3.58500D-03	-2.00990D-01	-2.93615D-02	8.57289D-01	-9.02132D-01
-1.00450D-01	1.01558D-02	-1.23576D-01	-1.62488D-01	-1.62399D-01	3.39264D-01
3.53549D-02	1.31408D-02	1.54478D-02	1.10806D-03	1.01947D-02	9.73011D-03
6.55427D-03	1.85608D-03	2.90773D-03	-1.75116D-03	1.09960D-03	2.35310D-03
3.99519D-04	-1.06646D-03	3.12522D-04	1.10439D-04	6.16728D-03	-7.63737D-04
-4.77258D-02	-3.26159D-02	-1.36740D-02	7.36619D-02	1.32837D-01	-6.67265D-02
-8.44073D-02	8.44531D-03	-1.73616D-03	9.54352D-02	1.52706D-01	-5.64510D-02
3.65309D-01	7.42899D-02	2.47079D-01	1.91783D-01	1.56344D-01	-3.98913D-02
1.93810D-01	-1.05861D-01	4.01683D-01	2.54645D-01	3.91698D-02	-1.08466D-01
2.30557D-03	-1.10135D-03	4.77520D-03	2.59443D-03	2.09693D-04	7.57053D-04
7.51256D-02	-1.13510D-01	2.42124D-01	1.97667D-01	1.70613D-02	-1.20829D-01
1.13353D-01	-9.41748D-02	2.64286D-01	1.76348D-01	2.73168D-02	-9.27220D-02
4.49831D-03	-3.32275D-03	1.00577D-02	5.95856D-03	5.46183D-04	3.60426D-03
-9.65587D-02	9.22555D-01	7.20239D-02	5.07472D-01	1.90620D-01	-1.38228D-01
6.15255D-01	-3.23849D-02	-5.77231D-02	3.49762D-01	2.42231D-01	-6.75486D-02
6.01533D-01	1.00000D+00	0.0	6.37576D-01	2.74857D-01	-5.55243D-02

-7.94500D-01	-8.20671D-01	-5.30038D-02	3.93876D-01
-5.92096D-01	-5.57166D-01	-5.76613D-02	3.93897D-01
-8.59539D-03	-8.51248D-03	3.74201D-04	1.97724D-02
-1.31871D-03	-1.53329D-03	-7.25229D-04	4.65406D-03
3.54510D-04	-4.62904D-05	-7.14630D-04	1.52329D-03
-4.32254D-02	-4.69223D-02	1.09564D-02	2.28251D-02
-2.93135D-02	-3.43887D-02	-1.59676D-02	2.09551D-02
-9.73261D-02	-9.79876D-02	7.51593D-02	1.12480D-01
-1.87588D-02	-2.67022D-02	1.33973D-01	-2.32599D-02

```
-6.08708D-02  2.25138D-03  1.50706D-03  7.79925D-04
-7.18483D-03 -1.65207D-02  9.42613D-02 -4.71189D-02
 4.48151D-03 -1.72681D-02  8.80749D-02 -2.40229D-02
 6.61978D-04 -4.07244D-03  3.20994D-03 -1.59628D-01
 2.72455D-02  7.18832D-04  7.74621D-01 -1.17182D-02
-3.14211D-02 -3.72423D-02 -2.20941D-01  2.47640D-02
-3.76396D-03 -1.76218D-02  5.51728D-01  4.70281D-02

THE DESIRED REDUCED STATE ORDER IS    5
THE SIGNIFICANT STATES ARE
     1         2         5         11        16
THE RM MATRIX IS
  1.000000D+00  0.0           0.0           0.0           0.0           0.0
  0.0           1.000000D+00  0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           1.000000D+00  0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           1.000000D+00  0.0           0.0           0.0
  0.0           0.0           0.0           1.000000D+00  0.0           0.0
  0.0           0.0           0.0           0.0           0.0           1.000000D+00
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0

  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           1.000000D+00  0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  1.000000D+00  0.0           0.0           0.0           0.0           0.0
  0.0           1.000000D+00  0.0           0.0           0.0           0.0
  0.0           0.0           1.000000D+00  0.0           0.0           0.0
  0.0           0.0           0.0           1.000000D+00  0.0           0.0
  0.0           0.0           0.0           0.0           0.0           1.000000D+00
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
  0.0           0.0           0.0           0.0           0.0           0.0
```

```
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             1.000000D+00
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
0.0             0.0             0.0             0.0
1.000000D+00    0.0             0.0             0.0
0.0             1.000000D+00    0.0             0.0
0.0             0.0             1.000000D+00    0.0

THE FULL SYSTEM EIGENVALUES ARE

EIGENVALUE #  1 IS -0.54023D+03 +J(  0.0          )
EIGENVALUE #  2 IS -0.15643D+03 +J(  0.0          )
EIGENVALUE #  3 IS -0.49694D+02 +J( -0.13992D+02  )
EIGENVALUE #  4 IS -0.49694D+02 +J(  0.13992D+02  )
EIGENVALUE #  5 IS -0.54120D+02 +J(  0.0          )
EIGENVALUE #  6 IS -0.42613D+02 +J(  0.0          )
EIGENVALUE #  7 IS -0.22589D+02 +J(  0.0          )
EIGENVALUE #  8 IS -0.21581D+02 +J( -0.25081D+01  )
EIGENVALUE #  9 IS -0.21581D+02 +J(  0.25081D+01  )
EIGENVALUE # 10 IS -0.14489D+02 +J(  0.0          )
EIGENVALUE # 11 IS -0.73493D+01 +J(  0.0          )
EIGENVALUE # 12 IS -0.36018D+01 +J(  0.0          )
EIGENVALUE # 13 IS -0.64705D+00 +J(  0.0          )
EIGENVALUE # 14 IS -0.13099D+01 +J(  0.0          )
EIGENVALUE # 15 IS -0.18616D+02 +J(  0.0          )
EIGENVALUE # 16 IS -0.19798D+01 +J(  0.0          )
```

THE TRANSPOSE MATRIX IS

-2.20100D+00	2.88800D-01	-7.13300D+02	-1.98900D+00	3.66900D-02	4.57800D+00
-2.50800D-01	-3.29900D+00	-2.89100D+02	-2.09000D+00	-2.48700D-01	1.25400D+02
2.30800D-03	-8.18400D-03	-9.18400D+00	-2.16400D-02	-1.01900D-02	1.03000D-02
-1.84900D+00	-8.25000D+00	-1.98400D+03	-4.93200D+01	-3.41200D+00	1.23700D+01
-1.05200D+00	1.96800D+00	4.61100D+02	-1.85400D+00	-4.88000D+01	7.31300D+00
6.32500D-01	3.28000D+00	7.10300D+02	2.95600D+00	8.27500D-01	-1.45100D+02
2.17000D-01	-6.86600D-03	5.05600D+01	2.59200D+00	-3.30200D-01	1.22200D+02
9.65400D-01	-6.18000D-02	7.46600D+01	5.96100D-02	1.19000D-01	1.34400D+00
6.31300D-01	-1.24100D-01	1.28600D+02	5.70300D-02	1.10100D-01	1.58700D+00
-1.58400D-01	1.04600D+00	-1.69500D+02	-1.38000D-01	-2.96000D-01	3.12400D+01
-1.00000D-02	-1.87800D-02	-1.17000D+01	3.93600D+01	-2.44600D-02	5.94500D-02
-1.50500D-04	-2.80000D-04	-1.73900D-01	5.83000D-01	-3.80500D-04	7.92700D-04
-2.43700D-02	-4.38400D-01	-2.85100D+01	3.85600D+01	-1.01900D-02	-1.12500D+02
-1.08700D-03	-1.94800D-02	-1.26900D+00	1.71400D+00	-4.89200D-04	-5.00200D+00
-1.61800D-01	-2.49600D-01	3.30100D+02	1.77500D+01	1.63100D-03	-5.15100D+01
-4.82200D-03	2.77900D-02	6.72400D+00	2.28100D-02	1.06000D-02	-1.18900D-02

4.07000D+02	-3.90200D+00	-5.67100D-01	-4.91300D-01	5.30300D+00	6.56900D+00
-2.44800D+02	1.60200D+01	-1.11300D+00	-1.46600D+00	5.48300D+00	4.44400D+00
1.44100D+00	7.55400D-02	-2.17400D-02	-1.38500D-02	-1.00900D-03	-4.38700D-02
-8.81100D+01	9.61600D+01	-1.04600D+01	3.41400D+01	-1.78000D+00	-1.61700D+01
2.70300D+02	-2.11000D+01	3.10500D+00	5.92100D-01	1.23100D+00	6.25100D+00
-2.05900D+01	-3.63500D+01	3.10800D+00	3.12700D+00	4.27200D-01	3.92200D+00
-5.47700D+02	-4.54600D+00	-3.73300D-01	-1.26100D+00	-1.54800D+00	-2.93700D+00
2.99300D+00	-1.88100D+01	4.32200D-01	1.08500D-01	6.35500D-02	5.64200D-01
8.98000D+00	2.37600D+00	-1.94300D+01	1.00500D-01	5.66800D-02	5.21700D-01
-7.56200D+00	2.62200D+01	-8.97300D-01	-2.02600D+01	-1.53700D-02	-1.40300D-01
-4.53100D-01	5.44100D-01	-7.17900D-02	6.62300D+00	-5.00100D+01	-1.15900D-01
-6.75200D-03	8.01900D-03	-1.10100D-03	9.81100D-02	-6.66800D-01	-6.68500D-01
4.17100D+02	1.55400D+00	-1.25600D-01	9.82000D+00	-9.81000D+00	-9.87400D+00
1.85400D+01	6.90200D-02	-5.59900D-03	4.36400D-01	-4.36000D-01	-4.39000D-01
4.20400D+01	1.35300D+00	-8.47100D-02	4.52700D+00	-4.51100D+00	-4.49800D+00
2.02600D-01	-3.06400D-01	1.99100D+01	1.17300D-01	7.72900D-03	5.02400D-02

4.71700D+00	4.87200D+00	-3.57100D-01	-1.35500D-01
-1.69000D-01	-1.54700D+00	-1.97000D-01	-6.13500D-01
-7.06400D-03	-6.39800D-02	2.19300D-02	4.21200D-03
-2.39600D+00	-2.17200D+01	-2.63500D+00	-7.24400D+00
8.30600D-01	7.56500D+00	2.94400D+01	2.46700D+01
5.75700D-01	5.28400D+00	6.46500D-01	1.91900D+00
-2.32000D-01	-2.06200D+00	-2.40100D-01	-5.20700D-01
8.47700D-02	7.63000D-01	9.54400D-02	2.76800D-01
7.44600D-02	7.11400D-01	8.92800D-02	3.29600D-01
-2.05100D-01	-1.84600D+00	-2.28400D-01	-6.27700D-01
-1.71800D-02	-1.41800D-01	-1.53900D-02	-3.92300D-02
-2.67300D-04	-2.23400D-03	-2.46300D-04	-6.18200D-04
-5.00200D+01	-2.19100D-01	-6.15700D-03	-6.06300D-02

```
 -2.001000D+00 -2.010000D+00 -3.079000D-04 -2.711000D-03
 -3.139000D+00 -3.253000D+00 -1.974000D+01 -3.424000D-02
  1.031000D-02  6.702000D-02  1.171000D-02 -1.997000D+01

THE T11 MATRIX IS
 -2.201000D+00  2.688000D-01 -7.133000D+02 -1.989000D+00  3.669000D-02
 -2.508000D-01 -3.299000D+00 -2.891000D+02 -2.090000D+00 -2.487000D-01
  2.308000D-03 -8.184000D-03 -9.184000D+00 -2.164000D-02 -1.019000D-02
 -1.849000D+00 -8.250000D+00 -1.984000D+03 -4.932000D+01 -3.412000D+00
 -1.052000D+00  1.968000D+00  4.611000D+02 -1.854000D+00 -4.880000D+01

THE T12 MATRIX IS
  4.578000D+00  4.070000D+02 -3.902000D+00 -5.671000D-01 -4.913000D-01  5.303000D+00
  1.254000D+02 -2.448000D+02  1.602000D+01 -1.113000D+00 -1.466000D+00  5.483000D+00
  1.030000D-02  1.441000D+00  7.554000D-02 -2.174000D-02 -1.385000D-02 -1.009000D-03
  1.237000D+01 -8.811000D+01  9.616000D+01 -1.046000D+01  3.414000D+01 -1.780000D+00
  7.313000D+00  2.703000D+02 -2.111000D+01  3.135000D+00  5.921000D-01  1.231000D+00

  6.569000D+00  4.717000D+00  4.872000D+00 -3.571000D-01 -1.355000D-01
  4.444000D+00 -1.690000D-01 -1.547000D+00 -1.979000D-01 -6.135000D-01
 -4.387000D-02 -7.064000D-03 -6.398000D-02  2.193000D-02  4.212000D-03
 -1.617000D+01 -2.386000D+00 -2.172000D+01 -2.635000D+00 -7.244000D+00
  6.251000D+00  8.306000D-01  7.565000D+00  2.944000D+01  2.467000D+01

THE T21 MATRIX IS
  6.325000D-01  3.280000D+00  7.103000D+02  2.956000D+00  8.275000D-01
  2.170000D-01 -6.866000D-03  5.056000D+01  2.592000D+00 -3.302000D-01
  9.654000D-01 -6.180000D-02  7.466000D+01  5.961000D-02  1.190000D-01
  6.313000D-01 -1.241000D-01  1.286000D+02  5.703000D-02  1.101000D-01
 -1.534000D-01  1.046000D+00 -1.695000D+02 -1.380000D-01 -2.960000D-01
 -1.000000D-02 -1.878000D-02 -1.170000D+01  3.936000D+01 -2.446000D-02
 -1.505000D-04 -2.800000D-04 -1.739000D-01  5.830000D-01 -3.805000D-04
 -2.437000D-02 -4.384000D-01 -2.851000D+01  3.856000D+01 -1.019000D-02
 -1.087000D-03 -1.948000D-02 -1.269000D+00  1.714000D+00 -4.892000D-04
 -1.618000D-01 -2.496000D-01  3.301000D+02  1.775000D+01  1.631000D-03
 -4.322000D-03  2.779000D-02  6.724000D+00  2.281000D-02  1.060000D-02

THE T22 MATRIX IS
 -1.451000D+02 -2.059000D+01 -3.635000D+01  3.108000D+00  3.127000D+00  4.272000D-01
  1.222000D+02 -5.477000D+02 -4.546000D+00 -3.733000D-01 -1.261000D+00 -1.543000D+00
  1.344000D+00  2.993000D+00 -1.881000D+01  4.322000D-01  1.085000D-01  6.355000D-02
  1.587000D+00  8.980000D+00  2.376000D+00 -1.943000D+01  1.005000D-01  5.668000D-02
  3.124000D+01 -7.562000D+00  2.622000D+01 -3.973000D-01 -2.026000D+01 -1.537000D-02
  5.945000D-02 -4.501000D-01  5.441000D-01 -7.179000D-02  6.623000D+00 -5.001000D+01
  7.927000D-04 -6.752000D-03  8.019000D-03 -1.101000D-03  9.811000D-02 -6.668000D-01
 -1.125000D+02  4.171000D+02  1.554000D+00 -1.256000D-01  9.820000D+00 -9.310000D+00
 -5.002000D+00  1.354000D+01  6.902000D-02 -5.599000D-03  4.364000D-01 -4.360000D-01
 -5.151000D+01  4.204000D+01  1.353000D+00 -3.471000D-02  4.527000D+00 -4.311000D+00
```

-1.139000D-02	2.026000D-01	-3.064000D-01	1.991000D+01	1.173000D-01	7.729000D-03
3.922000D+00	5.757000D-01	5.284000D+00	6.465000D-01	1.519000D+00	
-2.937000D+00	-2.320000D-01	-2.062000D+00	-2.401000D-01	-5.207000D-01	
5.642000D-01	8.477000D-02	7.630000D-01	9.544000D-02	2.768000D-01	
5.217000D-01	7.446000D-02	7.114000D-01	8.928000D-02	3.296000D-01	
-1.403000D-01	-2.051000D-01	-1.846000D+00	-2.284000D-01	-6.277000D-01	
-1.159000D-01	-1.718000D-02	-1.418000D-01	-1.539000D-02	-3.923000D-02	
-6.635000D-01	-2.673000D-04	-2.234000D-03	-2.463000D-04	-6.182000D-04	
-9.874000D+00	-5.002000D+01	-2.191000D-01	-6.157000D-03	-6.053000D-02	
-4.390000D-01	-2.001000D+00	-2.010000D+00	-3.079000D-04	-2.711000D-03	
-4.498000D+00	-3.199000D+00	-3.253000D+00	-1.974000D+01	-3.424000D-02	
5.024000D-02	1.031000D-02	6.702000D-02	1.170000D-02	-1.997000D+01	

```
THE REDUCED PLANT MATRIX IS
 -3.05092D+00  2.47257D+00 -3.58975D+02  8.69132D+00  9.18324D-02
  1.22428D-01 -1.62711D+00  4.10596D+01  4.28978D+00  3.02293D-01
  2.72857D-03 -2.83314D-03 -7.42764D+00 -1.16950D-02 -9.46242D-03
  4.13143D+00 -5.38539D+00 -1.62574D+03 -5.38526D+01 -2.79107D+00
 -1.07698D+00  2.23196D+00  1.14670D+03  1.90872D+01 -4.88967D+01

THE REDUCED CONTROL DISTRIBUTION MATRIX IS
 -3.54449D-02 -1.40804D+02 -9.33015D+01  2.35968D+01 -1.83786D+04
 -4.40945D-01 -2.72213D+01  7.83063D+00 -1.09597D+01 -9.97111D+03
  1.75264D-02 -3.76465D+01  1.21134D-01 -7.84650D-02 -1.19315D+02
  1.93016D+01  3.16353D+02  4.93851D+01 -7.68132D+01  4.83300D+04
 -1.89095D+00  7.27000D+01 -3.04357D+01  2.88674D+01  7.00352D+03

THE REDUCED OUTPUT DISTRIBUTION MATRIX IS
  1.92814D-01  1.07498D-01  1.87534D+02  1.20883D+00  7.07962D-02
  6.46066D-03 -9.94422D-07 -9.15428D-03 -5.67221D-05 -1.93162D-05
  0.0          0.0          0.0          1.00000D+00  0.0
  1.25351D-05  1.35123D-05 -3.12662D-02 -1.39005D-04 -4.84758D-05
 -4.88704D-05  1.47958D-04  1.01600D-03 -5.16836D-05  1.44907D-05
  1.63976D-05 -5.03547D-05 -3.75630D-02 -1.73080D-04 -5.95610D-05
  7.24150D-06 -8.12547D-07 -1.04928D-02 -4.94422D-05 -1.66866D-05

THE REDUCED FEEDFORWARD MATRIX IS
 -1.03766D-01 -3.14049D+01 -4.02029D+03 -3.99100D+00 -9.30000D+01
 -1.09289D-04 -2.27706D-01  3.46625D-01  4.91200D-03 -1.31400D+01
  0.0          0.0          0.0          0.0          0.0
  8.62091D-05 -7.01604D-03  1.83166D-02  5.68300D-07 -4.67100D-01
 -1.50784D-05  5.89803D-03  6.99583D-01 -2.74500D-03  2.70400D-01
  1.07941D-04  6.26683D-03  1.41085D-01 -6.40300D-04 -4.56000D-01
  2.92566D-05  1.14781D-03  4.63434D-02 -3.64200D-05 -1.29800D-01
```

OPTSYS4 SAMPLE OUTPUT

```
ORDER OF SYSTEM =  3

NUMBER OF CONTROLS =  5

NUMBER OF OBSERVATIONS =  7

NUMBER OF PROCESS NOISE SOURCES =  0

OPEN LOOP DYNAMICS MATRIX....

 -2.387D+00   1.601D+00  -6.255D+02
  4.521D-01  -2.056D+00  -8.753D+01
  1.763D-03  -1.721D-03  -7.176D+00
OPEN LOOP EIGENVALUES..
-3.282170D+00:-1.389470+00:-6.946740D+00:

RIGHT EIGENVECTOR MATRIX..

 9.507430D-01 -8.327950D-01  9.991800D-01
-3.099790D-01 -5.535810D-01  3.981450D-02
 5.673900D-04 -8.903110D-05  7.386320D-03

OPEN LOOP LEFT EIGENVECTOR MATRIX..
 7.471000D-01 -1.108630D+00 -9.508760D+01
-4.228340D-01 -1.180540D+00  6.356210D+01
-6.248610D-02  7.093110D-02  1.434560D+02

MEASUREMENT SCALING MATRIX

 2.856D-01  -1.333D-02   1.511D+02
 6.456D-03   4.572D-06  -7.629D-03
 7.331D-02  -1.003D-01  -3.078D+01
 1.551D-06   2.715D-05  -2.754D-02
-5.270D-05   1.532D-04   2.773D-03
 2.427D-06  -3.337D-05  -3.292D-02
 3.339D-06   4.040D-06  -9.162D-03
```

AD-A110 614   NAVAL POSTGRADUATE SCHOOL   MONTEREY CA   F/G 21/5

EFFECT ON FUEL EFFICIENCY OF PARAMETER VARIATIONS IN THE COST F--ETC(U)

SEP 81   B L DOUGHERTY

UNCLASSIFIED   NL

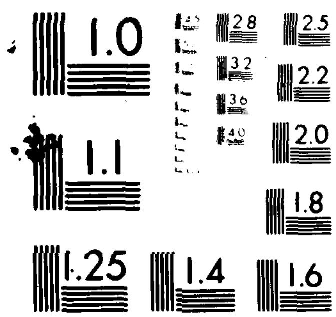
2 of 2

END

DATE FILMED

3 82

DTIC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

MODAL MEASUREMENT SCALING MATRIX

```
 3.6139690D-01 -2.4391970D-01  1.4009080D+00
 6.1322530D-03 -5.3783780D-03  6.3945360D-03
 3.6177840D-02 -5.2860610D-03 -1.5509700D-01
-2.2567240D-05 -1.3869470D-05 -2.0078860D-04
-3.6019550D-05 -4.1167180D-05 -2.6074530D-05
-5.5516520D-06  1.8966310D-05 -2.4156170D-04
-3.2762070D-06 -4.2014680D-06 -6.4176370D-05
```

MEASUREMENT FEEDTHROUGH MATRIX...

```
 3.416D-01  -3.030D+01  -2.932D+03  -3.991D+00  -9.300D+01
-1.318D-04  -2.277D-01   2.878D-01   4.912D-03  -1.314D+01
 3.624D-01   9.305D-01   8.724D+02   0.0         0.0
 3.085D-05  -7.133D-03  -1.264D-01   5.583D-07  -4.671D-01
-3.232D-05   5.846D-03   6.615D-01  -2.745D-03   2.704D-01
 3.910D-05   6.121D-03  -3.872D-02  -6.403D-04  -4.560D-01
 9.525D-06   1.106D-03  -4.861D-03  -3.642D-05  -1.298D-01
```

DIAGCNAL OUTPUT COST MATRIX...

```
1.000D-04   1.000D+00   [illegible].000D-04   1.000D+02   1.000D+02   1.000D-08
1.000D-08
```

THE CONTROL DISTRIBUTION MATR[illegible]X....

```
3.123D+00  -9.108D+01  -3.[illegible]4D+01   1.119D+01  -1.075D+04
1.145D+00  -1.725D+00   1.[illegible]4D+01  -1.708D+01  -6.083D+03
1.[illegible]32D-02  -3.775D+01   1.1[illegible]7D-01  -6.207D-02  -1.341D+02
```

MODAL CONTROL DISTRIBUTION MA[illegible]IX..

```
-1.0766570D-01  3.5234240D+03 -8.7414480D+01  3.3197510D+01  1.1463710D+04
-1.8891450D+00 -2.3589210D+03  2.9346310D+01  1.1486810D+01  3.2030180D+03
 1.6534470D+00 -5.4098370D+03  2.2326510D+01 -1.0815020D+01 -1.3997170D+04
```

```
THE CONTROL WEIGHTING MATRIX

   1.0000D-04   0.0          0.0          0.0          0.0
   0.0          1.0000D+02   0.0          0.0          0.0
   0.0          0.0          1.0000D-02   0.0          0.0
   0.0          0.0          0.0          1.0000D-02   0.0
   0.0          0.0          0.0          0.0          1.0000D-08

EIGENSYSTEM OF OPTIMAL CLOSED LOOP SYSTEM..

CLOSED LOOP OPTIMAL REGULATOR EIGENVALUES..
 -2.028000D+06:-1.102770D+05:-2.931100D+00:

RIGHT EIGENVECTOR MATRIX..

 -8.725450D-01 -8.623290D-01  5.533780D-02
 -4.883060D-01 -5.063410D-01  9.984680D-01
 -1.437600D-02  2.750860D-03 -7.646910D-05

CONTROL EIGENVECTOR MATRIX..

    3.6148530D-06  3.2262340D-06 -1.0015510D-03
   -6.8654790D-11 -5.6900810D-10  1.9943370D-07
   -2.2490450D-07 -3.0418430D-06  1.0082540D-03
   -2.4520690D-07  1.1901050D-06 -4.0151360D-04
   -1.6273290D+02 -9.1908000D+00  1.5173280D-04

CLOSED LOOP OPT. REG. LEFT EIGENVECTOR MATRIX..
 -1.840600D-01  5.364860D-03 -5.661300D+01
 -1.012120D+00  6.048920D-02  5.738020D+01
 -6.032820D-01  1.035080D+00  1.408710D+00
```

THE CONTROL GAINS ARE:

```
 6.0029D-04 -1.0365D-03 -1.4304D-03
-1.1973D-07  2.0639D-07  2.5218D-07
-6.0514D-04  1.0434D-03  1.2585D-03
 2.4107D-04 -4.1553D-04 -4.8344D-04
 3.9255D+01 -1.5102D+00  8.6864D+03
```

THE MODAL CONTROL GAINS ARE:

```
 8.911903D-04  7.397934D-05  5.479628D-04
-1.776639D-07 -1.457110D-08 -1.095479D-07
-8.980632D-04 -7.377931D-05 -5.538054D-04
 3.577229D-04  2.931190D-05  2.207541D-04
 4.271793D+01 -3.262855D+01  1.033230D+02
```

THE CLOSED LOOP DYNAMICS MATRIX IS..

```
-4.219913D+05  1.623607D+04 -9.337948D+07
-2.387864D+05  9.184462D+03 -5.283949D+07
-5.264065D+03  2.025152D+02 -1.164854D+06
```

CSMPIII SAMPLE PROGRAM

```
INITIAL
/       INTEGER*4 I,J,K
/       DIMENSION Q(5,16),A(16,16),B(16,5),C(7,16),D(7,5),
/      1E(16),F(16),G(7),H(7)
        STORAGE U(5),Y(7),R(2),T(5)
        FIXED I,J,K
        METHOD RKSFX
        RELERR X(1-16) = 1.0D-8
        ABSERR X(1-16) = 1.0D-8
        TABLE XI(1-16) = 50.,15*0.0D0
        TABLE U(1-5) = 5*0.D0
        TABLE Y(1-7) = 7*0.D0
        TABLE R(1-2) = 4052.9,4469.5
        TABLE T(1-5) = 150.,.2,2.0,4.0,0.0
NOSORT
 200    FORMAT(6E12.5)
        READ(5,200) ((A(I,J),J=1,16),I=1,16)
        READ(5,200) ((C(I,J),J=1,16),I=1,7)
        READ(5,200) ((D(I,J),J=1,5),I=1,7)
        READ(5,200) ((B(I,J),J=1,5),I=1,16)
        READ(5,200) ((Q(I,J),J=1,16),I=1,5)
DYNAMIC
NOSORT
       SFC=(R(1)+U(1))/(R(2)+Y(1))
       DO 90 I=1,5
       U(I)=0.D0
       DO 95 J=1,16
95     U(I)=U(I)+Q(I,J)*X(J)
       IF(U(I).LE.T(I)) GO TO 91
          U(I)=T(I)
91     IF(U(I).GE.(-T(I))) GO TO 90
          U(I)=-T(I)
90     CONTINUE
       DO 30 I=1,16
       E(I)=0.D0
       F(I)=0.D0
       DO 40 J=1,16
40     E(I)=E(I)+A(I,J)*X(J)
       DO 50 K=1,5
50     F(I)=F(I)+B(I,K)*U(K)
       XDCT(I)=E(I)+F(I)
30     CONTINUE
       DO 60 I=1,7
       G(I)=0.D0
       H(I)=0.D0
       DO 70 J=1,16
70     G(I)=G(I)+C(I,J)*X(J)
       DO 80 K=1,5
```

```
80      H(I)=H(I)+D(I,K)*U(K)
        Y(I)=G(I)+H(I)
60      CONTINUE
SORT
        X=INTGRL(XI,XDOT,16)
NOSORT
        OUTPUT X(1-16),U(1-5),Y(1-7),SFC
        TITLE F100 ENGINE SIMULATION
*       OUTPUT TIME,X(1),X(2)
*       PAGE XYPLOT
*       TITLE F100 CRUISE SIMULATION
*       OUTPUT TIME,Y(1)
*       PAGE XYPLOT
*       TITLE F100 CRUISE SIMULATION
*       OUTPUT TIME,SFC
*       PAGE XYPLOT
*       TITLE F100 CRUISE SIMULATION
TERMINAL
        TIMER FINTIM=3.0,OUTDEL=.05,PRDEL=.05,DELT=.001
        END
        INPUT
-0.22010D+01 0.28880D+00 0.45780D+01 0.40700D+03-0.71330D+03-0.39020D+01
-0.56710D+00-0.49130D+00 0.53030D+01 0.65690D+01-0.19890D+01 0.47170D+01
 0.48720D+01-0.35710D+00-0.13550D+00 0.36690D-01-0.25080D+00-0.32990D+01
 0.12540D+03-0.24480D+03-0.28910D+03 0.16020D+02-0.11130D+01-0.14660D+01
 0.54830D+01 0.44440D+01-0.20900D+01-0.16900D+00-0.15470D+01-0.19700D+00
-0.61350D+00-0.24870D+00 0.63250D+00 0.32800D+01-0.14510D+03-0.20590D+02
 0.71030D+03-0.36350D+02 0.31080D+01 0.31270D+01 0.42720D+00 0.39220D+01
 0.29560D+01 0.57570D+00 0.52840D+01 0.64650D+00 0.19190D+01 0.82750D+00
 0.21700D+00-0.68660D-02 0.12220D+03-0.54770D+03 0.50560D+02-0.45460D+01
-0.37330D+00-0.12610D+01-0.15430D+01-0.29370D+01 0.25920D+01-0.23200D+00
-0.20620D+01-0.24010D+00-0.52070D+00-0.33020D+00 0.23080D-02-0.81840D-02
 0.10300D-01 0.14410D+01-0.91840D+01 0.75540D-01-0.21740D-01-0.13850D-01
-0.10090D-02-0.43870D-01-0.21640D-01-0.70640D-02-0.63980D-01 0.21930D-01
 0.42120D-02-0.10190D-01 0.96540D+00-0.61900D-01 0.13440D+01 0.29930D+01
 0.74660D+02-0.18810D+02 0.43220D+00 0.10350D+00 0.63550D-01 0.56420D+00
 0.59610D-01 0.34770D-01 0.76300D+00 0.95440D-01 0.27680D+00 0.11900D+00
 0.63130D+00-0.12410D+00 0.15870D+01 0.89800D+01 0.12860D+03 0.23760D+01
-0.19430D+02 0.10050D+00 0.56680D-01 0.52170D+00 0.57030D-01 0.74460D-01
 0.71140D+00 0.89280D-01 0.32960D+00 0.11010D+00-0.15840D+00 0.10460D+01
 0.31240D+02-0.75620D+01-0.16950D+03 0.26220D+02-0.89730D+00-0.20260D+02
-0.15370D-01-0.14030D+00-0.13800D+00-0.20510D+00-0.18460D+01-0.22840D+00
-0.62770D+00-0.29600D+00-0.10000D-01-0.18780D-01 0.59450D-01-0.45010D+00
-0.11700D+02 0.54410D+00-0.71790D-01 0.60230D+01-0.50010D+02-0.11590D+00
 0.39360D+02-0.17130D-01-0.14180D+00-0.15390D-01-0.39230D-01-0.24460D-01
-0.15050D-03-0.28000D-03 0.79270D-03-0.67520D-02-0.17390D+00 0.30190D-02
-0.11010D-02 0.98110D-01-0.66080D+00-0.66350D+00 0.58300D+00-0.25730D-03
-0.22340D-02-0.24630D-03-0.61820D-03-0.38050D-03-0.18490D+01-0.32500D+01
```

0.123700D+02	-0.881100D+02	-0.198400D+04	0.961600D+02	-0.104600D+02	0.341400D+02
-0.178000D+01	-0.161700D+02	-0.493200D+02	-0.238600D+01	-0.217200D+02	-0.263500D+01
-0.724400D+01	-0.341200D+01	-0.243700D-01	-0.438400D+00	-0.112500D+03	0.417100D+03
-0.285100D+02	0.155400D+01	-0.125600D+00	0.982000D+01	-0.981000D+01	-0.987400D+01
0.385600D+02	-0.500200D+02	-0.219100D+00	-0.615700D-02	-0.606300D-01	-0.101900D-01
-0.108700D-02	-0.194800D-01	-0.500200D+01	0.185400D+02	-0.126900D+01	0.690200D-01
-0.559900D-02	0.436400D+00	-0.436000D+00	-0.439000D+00	0.171400D+01	-0.290100D+01
-0.201000D+01	-0.307900D-03	-0.271100D-02	-0.489200D-03	-0.161800D+00	-0.249600D+00
-0.515100D+02	0.420400D+02	0.330100D+03	0.135300D+01	-0.847100D-01	0.452700D+01
-0.451100D+01	-0.449800D+01	0.177500D+02	-0.319900D+01	-0.325300D+01	-0.197400D+02
-0.342400D-01	0.163100D-02	-0.482200D-02	0.277900D-01	-0.118900D-01	0.202600D+00
0.672400D+01	-0.306400D+00	0.199100D+02	0.117300D+00	0.772900D-02	0.502400D-01
0.228100D-01	0.103100D-01	0.670200D-01	0.117000D-01	-0.199700D+02	0.106000D-01
-0.105200D+01	0.196800D+01	0.731300D+01	0.270300D+03	0.461100D+03	-0.211000D+02
0.310500D+01	0.592100D+00	0.123100D+01	0.625100D+01	-0.185400D+01	0.830600D+00
0.755500D+01	0.294400D+02	0.246700D+02	-0.488000D+02		
0.369300D-01	-0.439100D+00	0.517500D+01	0.105600D+03	0.176900D+02	0.568100D+01
-0.279100D+00	-0.299800D+00	0.266900D+00	0.222600D+00	-0.105900D+01	-0.779000D-02
-0.824800D-01	0.186200D+01	0.137800D+01	-0.913100D-02	0.646800D-02	0.530100D-05
-0.130100D-03	-0.492300D-03	-0.706000D-02	-0.984500D-04	-0.314100D-04	-0.160400D-04
-0.939300D-05	-0.845300D-04	-0.804900D-05	-0.125300D-04	-0.112800D-03	-0.134700D-04
-0.312100D-04	-0.178300D-04	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.100000D+01	0.0	0.0	0.0	0.0	0.0
0.445100D-04	0.219000D-04	-0.443800D-04	-0.113900D-02	-0.258100D-01	-0.417100D-03
-0.135200D-03	-0.399700D-04	-0.230700D-04	-0.209800D-03	-0.208800D-04	-0.309400D-04
-0.281500D-03	-0.342100D-04	-0.939700D-04	-0.442600D-04	0.281500D-04	0.269100D-03
-0.555000D-02	0.134000D-02	0.301900D-01	-0.233200D-02	0.158700D-03	0.469100D-04
0.271100D-04	0.245800D-03	0.244800D-04	0.362300D-04	0.329900D-03	0.400500D-04
0.110300D-03	0.518200D-04	0.702600D-05	-0.322000D-04	-0.310000D-03	-0.154500D-02
-0.349400D-01	0.236200D-03	0.548700D-04	-0.539700D-04	-0.311900D-04	-0.284200D-03
-0.282100D-04	-0.422100D-04	-0.383700D-03	-0.464300D-04	-0.126600D-03	-0.599600D-04
0.723600D-05	0.565600D-05	-0.149200D-03	-0.418200D-03	-0.937800D-02	-0.124300D-04
0.416900D-04	-0.145900D-04	-0.836500D-05	-0.767500D-04	-0.760800D-05	-0.113600D-04
-0.103500D-03	-0.125300D-04	-0.344800D-04	-0.161900D-04		
0.341800D-01	-0.361400D+02	0.564300D+01	-0.399100D+01	-0.930000D+02	-0.112100D-03
-0.227500D+00	0.311300D+00	0.491200D-02	-0.131400D+02	0.0	0.0
0.0	0.0	0.0	0.779300D-04	-0.617600D-02	-0.365400D-02
0.568300D-06	-0.467100D+00	-0.836400D-04	0.786900D-02	0.103300D-02	-0.274500D-02
0.270400D+00	0.109100D-03	0.610300D-02	-0.573300D-03	-0.640300D-03	-0.456000D+00
0.285700D-04	0.115000D-02	0.329900D-03	-0.364200D-04	-0.129800D+00	
0.215700D-01	-0.223500D+03	-0.741000D+02	0.237500D+01	-0.277400D+03	0.567800D+00
-0.158200D+02	-0.813900D+01	-0.270700D+02	0.648300D+03	-0.151600D+01	0.656900D+02
0.220300D+02	0.329100D+02	-0.214600D+05	0.623200D+00	0.114200D+03	-0.264200D+01
0.414100D+00	-0.141400D+04	0.186800D-01	-0.379800D+02	0.115100D+00	-0.136700D+00
-0.761300D+02	-0.218400D+00	0.336600D+02	0.210200D+02	-0.841700D-01	0.155300D+04
-0.291000D+00	0.826000D+02	0.253000D+02	-0.532700D-01	0.199100D+04	0.513700D+00
-0.183400D+02	-0.610100D+01	0.368700D+01	-0.184200D+04	0.134900D+00	0.321500D+01

```
-0.46240D+00-0.54260D+00-0.61720D+04 0.20290D-02 0.57960D-01-0.70800D-02
-0.81870D-02-0.91460D+02 0.20970D+02 0.18130D+03-0.72980D+02-0.96020D+02
 0.50020D+05 0.29560D+00-0.16150D+01-0.89800D+00-0.12260D+01-0.67910D+04
 0.13160D-01-0.70400D-01-0.40240D-01-0.54610D-01-0.30180D+03 0.16030D+00
 0.59630D+01 0.15010D+00-0.93700D+00-0.29080D+04-0.24500D-01-0.80560D+00
-0.27420D+00 0.34770D+00 0.11930D+03-0.21520D+01-0.88430D+02-0.47430D+02
 0.24440D+02 0.12420D+05
-2.879100D-06-1.039800D-05 0.000000D+00 0.000000D+00 6.731000D-04 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00-1.367400D-05 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00 2.875500D-04 1.046900D-03 0.000000D+00 0.000000D+00
-6.893700D-02 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
-1.142800D-04-4.169000D-04 0.000000D+00 0.000000D+00 2.756900D-02 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 1.074200D+02-4.915200D+00
 0.000000D+00 0.000000D+00 5.523500D+04 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00 0.000000D+00
ENDINPUT
STOP
ENDJOB
```

LIST OF REFERENCES

1. Sain, M.K., and others, Alternatives for Linear Multivariable Control, NEC Inc., Chicago, Ill., 1978.

2. Szuch, J.R., "Techniques for Jet Engine Systems Modeling," Control and Dynamics, Volume 14, 1978.

3. Szuch, J.R. and Seldner, K., "A Real-Time Simulation of the F100-PW-100 Turbofan Engine Using the Hybrid Computer," NASA Technical Memo X-3261, NASA Lewis Research Center, 1975.

4. Pratt & Whitney Aircraft, Florida Research and Development Center, "F100-PW-100(3) Transient Engine Simulation Deck. User's Manual for Deck CCD 1103-3.0," FR-9476, January 1978.

5. Miller, R.J. and Hackney, R.D., "F100 Multivariable Control System Engine Models/Design Criteria," Air Force Aero Propulsion Laboratory, TR-76-74, 1976.

6. Kwakernaak, H. and Sivan, R., Linear Optimal Control Systems, John Wiley & Sons, New York, 1972.

7. Hall, W.E. and Bryson, A.E., "OPTSYS4 Computer Program," Stanford University, 1979.

8. DeHoff, R.L. and Hall, W.E., "State Space Techniques and Modeling for Control," Control and Dynamics, Vol. 14, 1978.

9. Weinberg, M.S. and Adams, G.R., "Low Order Linearized Models of Turbine Engines," Aeronautical Systems Division TR-75-24, July 1975.

10. DeHoff, R.L. and Skira, C.A., "A Practical Approach to Linear Model Analysis for Multivariable Turbine Engine Control Design," Alternatives for Linear Multivariable Control, NEC Inc., Chicago, Ill., 1978.

11. Kirk, D.E., Optimal Control Theory, Prentice Hall, Englewood Cliffs, N.J., 1970.

12. Hill, P.G. and Peterson, C.R., Mechanics and Thermodynamics of Propulsion, Addison-Wesley, Reading, Mass., 1965.

13. Speckhart, F.H. and Green, W.L., A Guide to Using CSMP, Prentice Hall, Englewood Cliffs, N.J., 1976.

14. Bryson, A.E. and Ho, Y.C., Applied Optimal Control, Hemisphere Publishing Co., Washington, D.C., 1975.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3.	Department Chairman, Code 67 Department of Aeronautics Naval Postgraduate School Monterey, California 93940	2
4.	Professor D.J. Collins, Code 67Co Department of Aeronautics Naval Postgraduate School Monterey, California 93940	1
5.	LT Barry Lawrence Dougherty, USN Box 73 East Meredith, New York 13757	1
6.	Mr. Charles A. Gardner Pacific Missile Test Center Point Mugu, CA 93042	1
7.	Dr. Gerhard Heiche AIR-310 Naval Air Systems Command Washington, DC 20361	1